Fully electromagnetic zonal flow residuals

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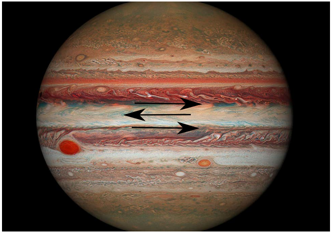
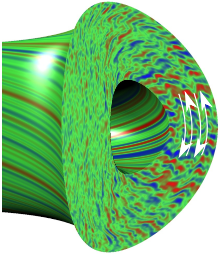
What are zonal flows?

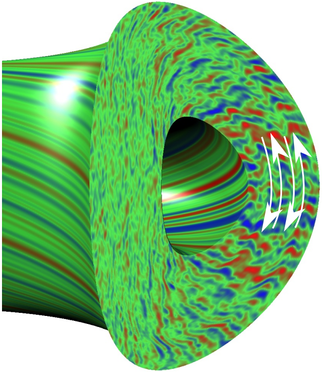
- **Turbulence driven sheared flows**, which influence turbulence.
- Planetary atmospheres: azimuthal symmetry, alternating in latitude. Also in accretion disks.
- Fusion plasmas: flux function, radially alternating Φ drives $\mathbf{E} \times \mathbf{B}$ flows.
- Sheared induced decorrelation of turbulent structures. **Transport regulation**.





Motivation

- Zonal flows strongly influence the transport level in a fusion reactor.
- What happens with zonal flows at finite β?
- Provide analytical benchmark for electromagnetic gyrokinetic codes.



Turbulent fluctuations in a tokamak; gyrokinetic simulation with GYRO. Arrows indicate zonal flows alternating across flux surfaces.

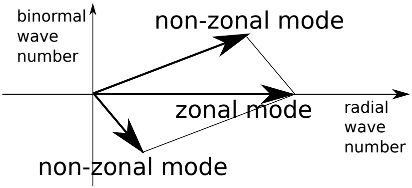
[Figures courtesy J. Candy and R. Waltz, General Atomics]

Overview

- The zonal flow residual
- Calculation: electromagnetic residual responses
- Predictions
- Gyrokinetic tests

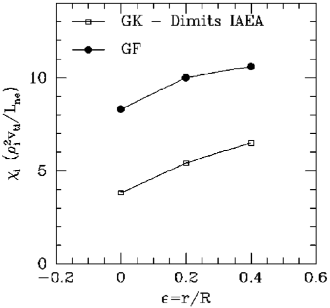
Zonal flows in fusion plasmas

- Flux function electrostatic fluctuations cannot drive radial transport, cannot tap into the gradient drive.
- Generated through non-linear interaction of finite toroidal mode number ($n \neq 0$) modes. ("Initial condition")
- $n \neq 0$ modes Landau damped; parallel electrostatic physics.
- Zonal modes (constant along **B**) are not Landau damped.



Realistic models needed for damping physics

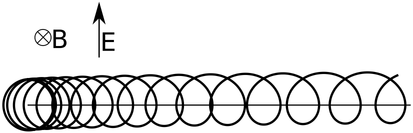
- Poloidal component of zonal flows damped.
- This damping is complete in fluid simulations due to unphysical Landau damping [Beer thesis 1995].
- Kinetically (without collisions) the poloidal part is reduced to a **finite residual** value, as modified by plasma polarization [Rosenbluth & Hinton 1997].
- **More residual zonal flow, more effective turbulence suppression!**



Turbulent ion heat diffusivity overestimated by gyrofluid (GF) models compared to gyrokinetic (GK). [Dimits 2000 PoP]

The physics of the residual

- Radial electrostatic perturbation induces plasma polarization; electric field gets partially shielded.



- Larmor motion → **classical polarization**.
  Guiding centre orbit → **neoclassical polarization**.
- Zonal mode excited between these time scales.
- Over several bounce times collisionless residual value reached

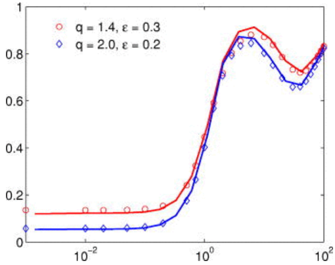
$$\frac{\Phi(t=\infty)}{\Phi(t=0)} = \frac{\text{classical pol.}}{\text{classical+neoclassical pol.}} \approx \frac{1}{1+1.64\frac{q^2}{\sqrt{\epsilon}}}.$$

(circular tokamak, long wave length)

- Plasma polarization dominated by trapped ion orbit motion.

The zonal flow residual

- The electrostatic long wavelength result by Rosenbluth and Hinton is a **standard test of gyrokinetic codes**.
- Later generalized to
  collisional plasmas [Hinton 1998]
  shaped plasmas [Xiao 2006/I PoP],
  arbitrary wave lengths [Xiao 2006/II PoP],
  pedestal [Landreman 2010 PPCF],
  stellarators [Sugama 2005 PRL],
  stell. + arb. wave length [Monreal 2016 PPCF].
- Externally imposed magnetic perturbations [Terry 2013 PoP]
- **Generalization to the electromagnetic case** with self-consistent magnetic fields **has been lacking**.



Electrostatic zonal flow residual $\frac{\Phi(t=\infty)}{\Phi(t=0)}$, as a function of $k_\perp \rho_i$.
Lines: analytical
Symbols: gyrokinetic simulations results with GS2.
[Xiao 2007 PoP]

Calculating the electromagnetic residual

- Choose $f_1(t=0)$ (Warning: Arbitrariness!)
- Evolve f_1 with Vlasov equation to long times
  (distribution independent of particle location along orbit)
- Evaluate fields at $t=0$ and $t\to\infty$ using Maxwell's equations
- Simplify expressions for final-to-initial field amplitude ratios

Perturbed fields

- **Fields are eikonal in radius** (ψ)

$$\{\Phi, A_\parallel, B_\parallel\} = \{\tilde{\Phi}, \tilde{A}_\parallel, \tilde{B}_\parallel\} e^{iS}$$

$\nabla S = S' \nabla \psi = k_\perp \mathbf{e}_\perp$

- $B_\parallel$ and $A_\parallel$ correspond to the compressional and shear magnetic perturbations.
- The coefficients $\{\tilde{\Phi}, \tilde{A}_\parallel, \tilde{B}_\parallel\}$ can be slow functions of ψ and θ.
- Assume **toroidal symmetry preserved** during the evolution

The kinetic equation

- No collisions, no background **E**
- Vlasov equation linearized around a Maxwellian background f_0

$$\dot{h} = \frac{\partial h}{\partial t} + \mathbf{v}\cdot\nabla h + \Omega \mathbf{v}\times\mathbf{b}\cdot\nabla_v h = \partial_t \left[\Phi - \frac{\mathbf{v}\cdot\mathbf{A}}{c}\right]\frac{Ze}{T} f_0,$$

$h = f_1 + Ze\Phi f_0/T$

- $\mathbf{R} = \mathbf{r} + \frac{1}{\Omega}\mathbf{v}_\perp \times \mathbf{b}$ for time scales longer than gyration
- **Use canonical angular momentum**

$$\psi_* = \psi + \frac{1}{\Omega}\mathbf{v}_\perp \times \mathbf{b}\cdot\nabla\psi - \frac{I v_\parallel}{\Omega}$$

for time scales longer than orbit motion.

- $$h = h(\psi_*, \theta, v, \mu, \varphi, t) = \tilde{h}_*(\psi_*, \theta, v, \mu, t) e^{iS(\psi_*)}$$

$\tilde{h}_*$ is slow function of ψ_* and θ

Obtaining long time behavior

- Long wavelength limit: $S(\psi) - S(\psi_*) \approx Q - L$

$$L = \frac{k_\perp v_\perp}{\Omega} \sin\varphi \sim k_\perp \rho \qquad Q = \frac{k_\perp v_\parallel}{\Omega_{pol}} \sim k_\perp \rho \frac{B}{B_\theta} \ll 1$$

gyro-motion and orbit motion across perturbations.

- Gyro-averaged kinetic equation

$$\partial_t \tilde{h}_* + v_\parallel \mathbf{b} \cdot \nabla \tilde{h}_* = \frac{Zef_0}{cT} e^{iQ} \partial_t \left[J_0 (c\tilde{\Phi} - v_\parallel \tilde{A}_\parallel) + \frac{J_1}{z} \frac{v_\perp^2}{2\Omega} \tilde{B}_\parallel \right],$$

J_0 and J_1 have the argument $z = \frac{k_\perp v_\perp}{\Omega}$

- $\tilde{h}_*(t \to \infty)$ is constant along particle orbits.
  Annihilate orbit motion, and integrate in time

$$\overline{\tilde{h}_*(t)} = \overline{\tilde{h}_*(0)} + \frac{Zef_0}{cT} \overline{e^{iQ} [\ldots(t)]} - \frac{Zef_0}{cT} \overline{e^{iQ} [\ldots(0)]},$$

Obtaining the fields

- Quasineutrality, $\sum Ze \int d^3v f_1 = 0$

$$\sum Ze \int d^3v\, \tilde{h}_* J_0 e^{-iQ} - \sum \frac{Z^2 e^2 n}{T} \tilde{\Phi} = 0,$$

- Parallel Ampère's law

$$k_\perp^2 \tilde{A}_\parallel = \frac{4\pi}{c} \sum Ze \int d^3v\, \tilde{h}_* v_\parallel J_0 e^{-iQ}$$

- Perpendicular Ampère's law (pressure balance)

$$\frac{B_0 \tilde{B}_\parallel}{4\pi} + \sum \int d^3v\, \tilde{h}_* \frac{M v_\perp^2}{2} \frac{2J_1}{z} e^{-iQ} = 0.$$

- Use $\tilde{h}_*(0)$ and $\overline{\tilde{h}_*(t)}$ to calculate initial and final fields.
- Do not assume fields to be flux functions.

Choice of initial condition

- **Use initial value formulation**.

$$\frac{\tilde{\Phi}(\infty)}{\tilde{\Phi}(0)} = \frac{1}{1 + 1.64\frac{q^2}{\sqrt{\epsilon}}} \quad \text{assumes} \quad \tilde{f}_1(0) \propto \frac{e\tilde{\Phi}(0)}{T_i} f_0$$

  - Why would nonlinearity produce exactly this distribution?
  - Different $\tilde{f}_1(0)$ can result in different residual values.
  - Still, representative of typical behavior and gives insights.
- **Choice of electromagnetic initial condition**
- $\tilde{h}_*(0)$ satisfies Maxwell's eqs. and gyro-phase independent.
- Terms in $\tilde{h}_*(0)$ parametrized by $\tilde{\Phi}^{(0)}$, $\tilde{A}_\parallel^{(0)}$ and $\tilde{B}_\parallel^{(0)}$ produce these flux function fields only at $t = 0$.
- Recover above electrostatic residual.
- **Simplest form** for $\tilde{h}_*(0)$ we can find.

Choice of initial condition

$$\tilde{h}_*(0) = \frac{Zef_0 e^{iQ}}{TJ_0}\left[H\tilde{\Phi}^{(0)} + \frac{k_\perp^2 c}{\omega_p^2} v_\parallel \tilde{A}_\parallel^{(0)} - u\frac{2T}{ZeB_0\beta}\tilde{B}_\parallel^{(0)}\right]$$

- Both H and u include velocity integrals.
- H has a velocity structure such that $\tilde{\Phi}^{(0)}$ term contains no pressure perturbation at $t = 0$.
- u relies on unperturbed quasineutrality to not contribute to $\tilde{\Phi}$ at $t = 0$.

Evaluation of residuals

- Circular cross section, large aspect ratio $\epsilon \ll 1$, and long wavelength $k_\perp \rho \ll k_\perp \rho \frac{B}{B_\theta} \ll 1$, allow finite Shafranov shift.
- The electrostatic residual field is approximately a flux function.
- **Magnetic residual fields need not be flux functions**, even if they are at $t = 0$. $\rightarrow$ Keep the $\cos\theta$ component.
- $\tilde{A}_\parallel$ and $\tilde{B}_\parallel$ need to be kept under orbit averages.
- **Electron dynamics cannot be neglected**; contribution to currents and pressure. Test of codes with kinetic electrons.
- Both trapped and circulating responses matter.

Obtaining residual response tests

- $$\langle \text{Quasineutrality} \rangle$$
  $$\langle \text{Ampère's law} \rangle$$
  $$\oint d\theta \cos\theta [\langle \text{Ampère's law} \rangle - \text{Ampère's law}]$$
  to get $\langle \tilde{\Phi} \rangle$, $\langle \tilde{A}_\parallel \rangle$, $\langle \tilde{B}_\parallel \rangle$,
  and the $\cos\theta$ components of $\tilde{A}_\parallel$ and $\tilde{B}_\parallel$: a and b.
- Linear equations in the unknowns, but complicated.
- Use assumed orderings to get leading order expressions for 15 residual responses:

$$\frac{\tilde{\Phi}}{\tilde{\Phi}^{(0)}}, \frac{\tilde{A}_\parallel}{\tilde{\Phi}^{(0)}}, \frac{a}{\tilde{\Phi}^{(0)}}, \frac{\tilde{B}_\parallel}{\tilde{\Phi}^{(0)}}, \frac{b}{\tilde{\Phi}^{(0)}}, \ldots \frac{\tilde{\Phi}}{\tilde{A}_\parallel^{(0)}}, \ldots \frac{b}{\tilde{B}_\parallel^{(0)}}$$

Skin depth

- **At skin depth scale reconnection happens** without collisions.

$$\left(\frac{\text{skin depth}}{\text{wave length}}\right)^2 \sim \frac{(k_\perp \rho_i)^2}{\beta} \frac{m_e}{m_i}$$

- We allow for β to be so small to make the skin depth large.
- But skin depth is usually small, as in following comparisons.

FIVE YEARS LATER

[Catto, Parra and Pusztai 2017 JPP] Errata pending

Electrostatic initial condition

- Only $\tilde{\Phi}^{(0)} \neq 0$
- Relative modification of electrostatic residual $\tilde{\Phi}/\tilde{\Phi}^{(0)}$ is small

$$\frac{\langle\tilde{\Phi}\rangle}{\tilde{\Phi}^{(0)}} = \frac{1}{1+\frac{\gamma q^2}{\sqrt{\epsilon}}}\left(1+\mathcal{O}\left(\frac{\beta(k_\perp \rho_i)^2}{\sqrt{\epsilon}}\right)\right)$$

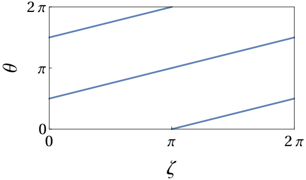
- The distribution develops a pressure perturbation that is balanced by a compressional magnetic perturbation

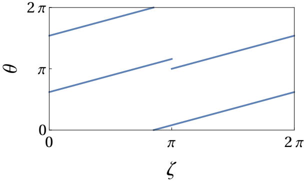
$$\frac{v_i \tilde{B}_\parallel}{k_\perp c \tilde{\Phi}^{(0)}} \sim \beta k_\perp \rho_i$$

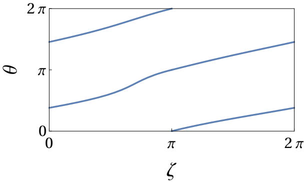
- Relative poloidal variation of $\tilde{B}_\parallel$ is $\mathcal{O}(1)$.

Constraints on $\tilde{A}_\parallel$ in small skin depth limit

- Change in $\tilde{A}_\parallel$ modifies field line pitch
- Change in field line topology requires reconnection
- Small skin depth, no collisions $\Rightarrow$ **no reconnection happens**.
- $\Delta\tilde{A}_\parallel \neq 0$ is still possible but **should not affect safety factor**.
- In our geometry, $\tilde{A}_\parallel$ cannot develop any θ-independent component. $\cos\theta$ component still contributes to flux surface average.







Electrostatic initial condition, contd.

- $\tilde{A}_\parallel$ has no θ-independent component

$$\frac{v_i \tilde{A}_\parallel}{c\tilde{\Phi}^{(0)}} \sim i\beta k_\perp \rho_i \cos\theta$$

- If we assumed $\tilde{A}_\parallel$ to be a flux function it would be $\left(\frac{\text{skin depth}}{\text{wave length}}\right)^2$ smaller!

Shear magnetic initial condition

- Only $\tilde{A}_\parallel^{(0)} \neq 0$
- The θ-independent part of $\tilde{A}_\parallel^{(0)}$ cannot change with time.
- But $A_\parallel$ can still develop some poloidal variation:

$$\tilde{A}_\parallel = \tilde{A}_\parallel^{(0)}(1 - \epsilon \cos\theta)$$

$$\Rightarrow \qquad \frac{\langle \tilde{A}_\parallel \rangle}{\tilde{A}_\parallel^{(0)}} = 1 - \frac{\epsilon^2}{2}$$

- $\tilde{A}_\parallel^{(0)}$ induces electrostatic and compressional magnetic residuals:

$$\frac{c\tilde{\Phi}}{v_i \tilde{A}_\parallel^{(0)}} \sim -i\epsilon k_\perp \rho_i, \qquad \frac{\tilde{B}_\parallel}{k_\perp \tilde{A}_\parallel} \sim -i\epsilon\beta$$

Compressional magnetic initial condition

- Only $\tilde{B}_{\parallel}^{(0)} \neq 0$
- $\frac{\delta n}{n} \sim \frac{\delta p}{p}$ and a Boltzmann size electron response yields

$$\frac{e\delta\Phi}{T} \sim \frac{1}{\beta}\frac{\delta B}{B}$$

- $\tilde{B}_{\parallel}^{(0)}$ **generates sizable** $\tilde{\Phi}$, consistent with above estimate

$$\frac{ck_{\perp}\tilde{\Phi}}{v_i\tilde{B}_{\parallel}^{(0)}} \sim \frac{k_{\perp}\rho_i}{\beta}$$

- In essentially electrostatic turbulence non-zonal compressional magnetic fluctuations scale with β.
  Importance of the $\tilde{\Phi}$ response to $\tilde{B}_{\parallel}^{(0)}$ is uncertain.

Compressional magnetic initial condition

- $\tilde{B}_\parallel$ remains close to $\tilde{B}_\parallel^{(0)}$.

$$\frac{\langle \tilde{B}_\parallel \rangle}{\tilde{B}_\parallel^{(0)}} - 1 \sim -\beta$$

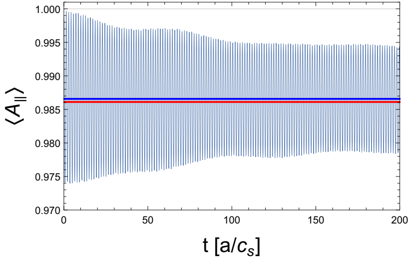
- Compressing field lines leads to rearrangement of field line pitch on flux surface

$$\frac{k_\perp \tilde{A}_\parallel}{\tilde{B}_\parallel^{(0)}} \sim i \cos\theta$$

- q is unaffected by $\tilde{A}_\parallel$.

Gyrokinetic tests with CGYRO

- Implemented initial conditions in the flux-tube continuum gyrokinetic code CGYRO [Belli 2016 J. Comp. Phys.].
- Scans around baseline parameters: $\beta_e = 0.003$, $R/a = 3$, $r/a = 0.5$, $q = 2$, $s = 1$, $\nu_{ee} = 0$, $k_\perp \rho_s = 0.0377$, $T_e = T_i$ deuterium plasma, no profile gradients, physical mass ratio.
- Filed components are time averaged values.



Time series of $\mathrm{Re}\langle A_\parallel \rangle$ from CGYRO simulation with $A_\parallel(0) = 1$, together with time average, prediction.

Gyrokinetic tests with CGYRO

- One of the three terms is non-zero in

$$\tilde{h}_*(0) = \frac{Zef_0 e^{iQ}}{TJ_0}\left[H\tilde{\Phi}^{(0)} + \frac{k_\perp^2 c}{\omega_p^2} v_\parallel \tilde{A}_\parallel^{(0)} - u\frac{2T}{ZeB_0\beta}\tilde{B}_\parallel^{(0)}\right],$$

corresponding field component is 1 in CGYRO units

- Normalizations:

$$\Phi = \frac{e\tilde{\Phi}}{T_e}, \quad A_\parallel = \frac{c_s e\tilde{A}_\parallel}{cT_e}, \quad B_\parallel = \frac{\tilde{B}_\parallel}{B_u},$$

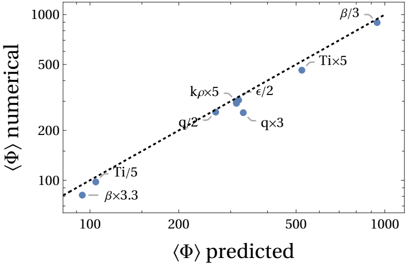
- Current baseline case is not asymptotically small in ϵ and $\frac{q}{\epsilon}k_\perp\rho_s$. Sometimes agreement is OK for scalings but not numerically accurate.

Compressional magnetic initial condition, Φ

$$\Phi \approx \frac{2}{\beta_{eu}\left(1+\frac{ZT_e}{T_i}\right)\left(1+\frac{\sqrt{\epsilon}}{\gamma q^2}\right)}$$

$$\gamma \approx 1.64$$

- Response is large in $1/\beta$
- Φ remains close to a flux function in all cases.

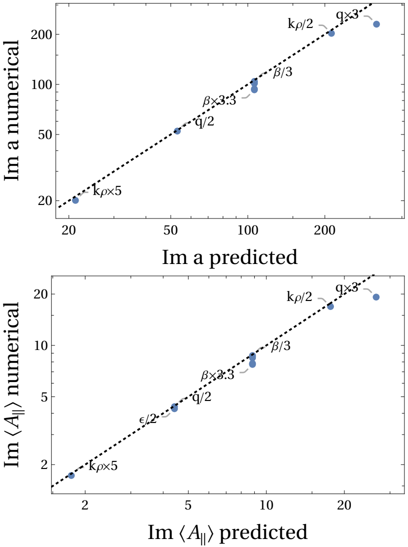


- T_i, q, $k_\perp \rho_s$, ϵ and β_{eu} are changed around baseline values.

Compressional magnetic initial condition, $A_\parallel$

$$\mathrm{Im}\, a = \frac{2q}{k_\perp \rho_s}, \quad \Rightarrow \mathrm{Im}\langle A_\parallel \rangle = \frac{q\epsilon}{k_\perp \rho_s}$$

- Response is large in $1/(k_\perp \rho_s)$
- Reasonable agreement between simulation and predictions
- Average value in simulation does not drift for a long time (unless collisions are switched on): **No significant reconnection of numerical origin.**

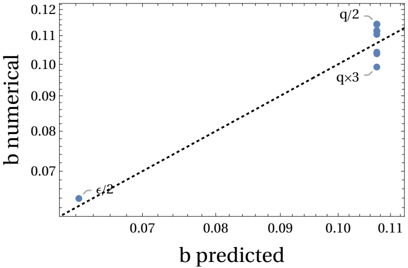


Compressional magnetic initial condition, $B_\parallel$

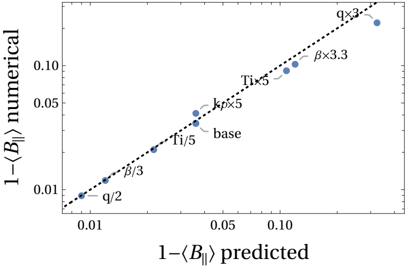
$$b = \epsilon\left(1 - \frac{\varsigma\sqrt{\epsilon}}{5}\right),$$

$$\langle B_\parallel\rangle = 1 - \frac{3}{2}q^2\beta_{eu}\left(1 + \frac{T_i}{ZT_e}\right)$$

$$\varsigma \approx 4.36$$



- $\sqrt{\epsilon}$ correction in b is important to keep.
- $\langle B_\parallel\rangle$ remains close to its initial value, difference is well reproduced
- Interestingly, contribution of b to $\langle B_\parallel\rangle$ cancels out.



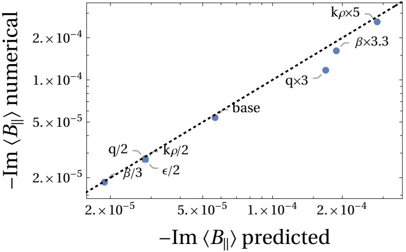
Shear magnetic initial condition, $B_\parallel$

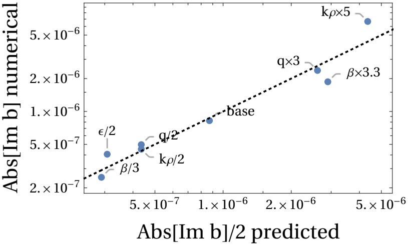
$$\mathrm{Im}\left\langle B_\parallel \right\rangle \approx -\frac{3}{4} q\epsilon k_\perp \rho_s \beta_{eu} \left(1 + \frac{T_i}{ZT_e}\right)$$

$$\mathrm{Im}\, b \approx \frac{\chi}{4} q\epsilon^{3/2} k_\perp \rho_s \beta_{eu} \left(1 + \frac{T_i}{ZT_e}\right),$$

$$\chi \approx 0.2262$$

- $\mathrm{Im}\left\langle B_\parallel \right\rangle$ is well reproduced.
- Competition between trapped and passing contributions changes the sign of b at finite ϵ.





Conclusions

- Initial value calculation of collisionless axisymmetric zonal flow residual generalized to include electromagnetic perturbations.
  - As a response to pure $B_\parallel$ initial perturbation a Φ response large in $1/\beta$ can develop.
  - Poloidal variations in $A_\parallel$ and $B_\parallel$ are important, while Φ remains close to a flux function.
  - Assuming $A_\parallel$ to not have poloidal variation would lead to a response small in $\left(\frac{\text{skin depth}}{\text{wave length}}\right)^2$.
  - Even in the small skin-depth limit $A_\parallel$ may be finite, but it corresponds to a redistribution of field line pitch on a flux surface such that q remains unperturbed.
- Can be used as a stringent test of fully electromagnetic gyrokinetic codes with kinetic electrons.
- Comparisons with Eulerian flux-tube code CGYRO.

Extra slides

Initial condition

$$\tilde{h}_*(0) = \frac{Zef_0 e^{iQ}}{TJ_0}\left[H\tilde{\Phi}^{(0)} + \frac{k_\perp^2 c}{\omega_p^2} v_\| \tilde{A}_\|^{(0)} - u\frac{2T}{ZeB_0\beta}\tilde{B}_\|^{(0)}\right]$$

- $$H = 1 - \left(\frac{mv^2}{2T} - \frac{3}{2}\right)\frac{\int d^3v\, f_0 v_\perp^2\left(\frac{2J_1}{zJ_0} - 1\right)}{\int d^3v\, f_0 v_\perp^2 \frac{2J_1}{zJ_0}\left(\frac{mv^2}{2T} - \frac{3}{2}\right)},$$
- $$u = \frac{\sum nT}{\sum \int d^3v\, f_0 \frac{mv_\perp^2}{2}\frac{2J_1}{zZ_0}},$$

  is species independent.

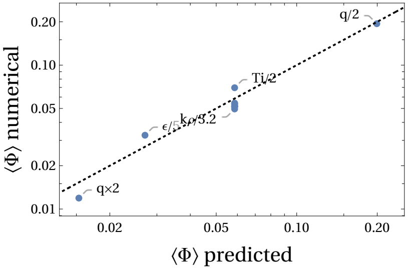
Perturbation of q

- Split safety factor into unperturbed and perturbed parts $q = q_0 + q_1$.
- Define θ such that it is unaffected the perturbation, and use that $k_\perp / RB_\theta = S'$ is independent of θ.
- In large aspect ratio circular geometry

$$q_1 = \frac{1}{2\pi} \frac{ik_\perp}{RB_\theta} \int_0^{2\Pi} \frac{A_\parallel}{\mathbf{b} \cdot \nabla\theta} d\theta$$

Electrostatic initial condition, Φ

$$\Phi \approx \frac{1}{1+\frac{\gamma q^2}{\sqrt{\epsilon}}} \left\{1 + \mathcal{O}\left[\beta_{eu}\epsilon^{3/2}(k_\perp \rho_s)^2\right]\right\}$$



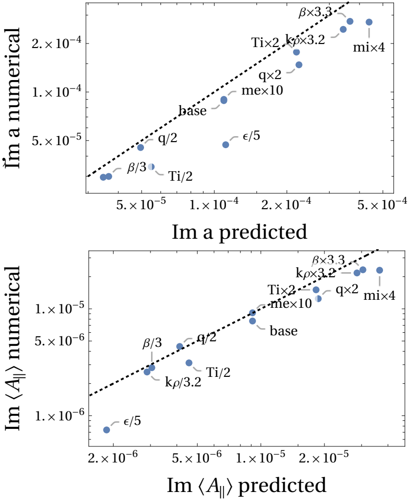
- The leading order correction to the electrostatic result is too small to be used for a comparison.

Electrostatic initial condition, $A_\parallel$

$$\mathrm{Im}\, a = \frac{1}{2} q \beta_{eu} k_\perp \rho_s \frac{m_i T_i}{m_D T_e Z^2}\left(1 - \frac{\sqrt{\epsilon}}{2\gamma q^2}\right)$$

$$\Rightarrow \mathrm{Im}\,\langle A_\parallel \rangle = \frac{a\,\epsilon}{2}$$

- Result consistent with no magnetic reconnection



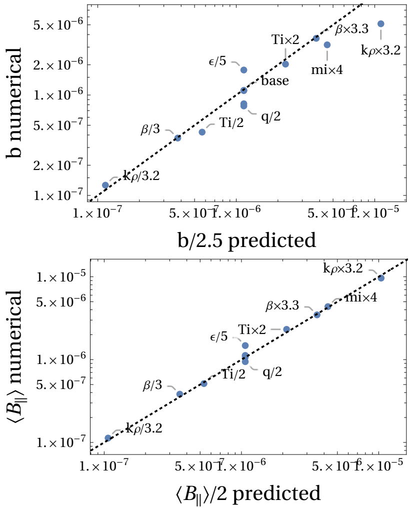
Electrostatic initial condition, $B_\parallel$

$$b = \frac{\varsigma}{4\gamma}\beta_{eq}(k_\perp \rho_s)^2 \frac{m_i T_i}{Z^2 m_D T_e},$$

$$\langle B_\parallel \rangle = \frac{1}{2}\beta_{eq}(k_\perp \rho_s)^2 \frac{m_i T_i}{Z^2 m_D T_e},$$

where $\varsigma \approx 4.36$

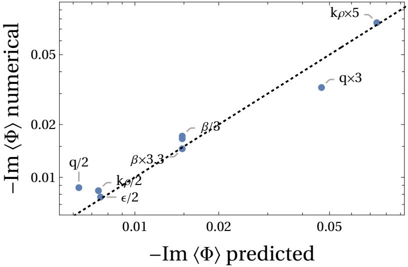
- Parametric scalings are fine, but results differ by $\mathcal{O}(1)$ factors.



Shear magnetic initial condition, Φ

$$\mathrm{Im}\,\Phi = -\frac{5}{4} k_\perp \rho_s \, q \, \epsilon \frac{T_i}{Z T_e} \frac{1}{1 + \frac{\sqrt{\epsilon}}{\gamma q^2}}$$

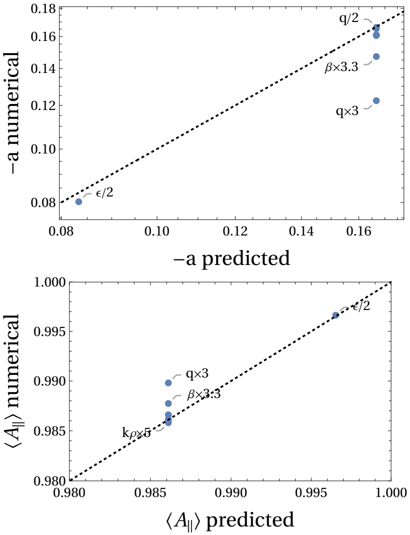
- Response is small in $k_\perp \rho_s \, \epsilon$
- Change $k_\perp \rho_s$, q, β_{eu} and ϵ around baseline values



Shear magnetic initial condition, $A_\parallel$

$$a = -\epsilon, \quad \Rightarrow \langle A_\parallel \rangle = 1 - \frac{\epsilon^2}{2}$$

- $A_\parallel$ remains close to its initial value, while developing a $\sim \epsilon$ poloidal variation.
- Results consistent with no magnetic reconnection
- at high q the agreement is not as good as $k_\perp \rho_{i\mathrm{pol}}$ is not sufficiently small anymore.



Sketch of the electrostatic residual calculation

- Consider the dynamics of toroidally symmetric $(n = 0)$ modes.
- Effect of nonlinear excitation of zonal modes represented by time dependent sources Υf_0.
- Solve the electrostatic gyrokinetic equation for toroidally symmetric perturbations

$$\left[\partial_t + v_\parallel \mathbf{b} \cdot \nabla + i\omega_D\right] g = \frac{Ze}{T} f_0 J_0 \frac{\partial \Phi}{\partial t} + \Upsilon f_0$$

where the non-adiabatic response $g = f_1 + \frac{Ze\Phi}{T} f_0$ depends on guiding centre position $\mathbf{R}$.

Sketch of the electrostatic residual calculation

$$\left[\partial_t + v_\parallel \mathbf{b}\cdot\nabla + i\omega_D\right] g = \frac{Ze}{T} f_0 J_0 \frac{\partial \Phi}{\partial t} + \Upsilon f_0$$

- Radial eikonal modes $\Phi = \tilde{\Phi} e^{iS}$ with $\nabla S = S'(\psi)\nabla\psi = \mathbf{k}_\perp$.
- Interested in long time asymptotic behavior, where orbit motion dominates. $\left[v_\parallel \mathbf{b}\cdot\nabla + i\omega_D\right]\tilde{g_0} = 0 \Rightarrow \tilde{g_0} = Ge^{-iQ}$.
- $Q = k_\perp v_\parallel/\Omega_p$. Analogous to $k_\perp v_\perp/\Omega$, but describes radial drifts (instead of gyro-motion) across perturbations.
- Multiply next order equation by e^{iQ}, orbit average, and integrate in time to obtain the flux function, G.

$$G(t) = \overline{e^{iQ}\frac{Ze}{T} f_0 J_0 \Phi(t)} + \overline{e^{iQ} f_0 \int_{-\infty}^{t} dt' \Upsilon(t')}$$

Sketch of the electrostatic residual calculation

$$G(t) = \overline{e^{iQ}\frac{Ze}{T}f_0 J_0 \Phi(t)} + \overline{e^{iQ} f_0 \int_{-\infty}^{t} dt' \Upsilon(t')}$$

- Use $\tilde{g}_0 = Ge^{-iQ}$ in quasineutrality to get $\Phi(t)$. Leads to an integral equation; can be solved using Laplace transform.
- In general $\tilde{\Phi}(t) = \int_{-\infty}^{t} dt' \mathcal{K}(t-t')\Upsilon(t')$.
- The electrostatic zonal flow residual is the long time asymptotic value of: $\frac{\tilde{\Phi}(t=\infty)}{\tilde{\Phi}(t=0)}$
- Take $\Upsilon \propto \delta(t)$, and relate Υ to $\Phi(0)$ assuming that that classical polarization has happened already at $t = 0$.
- Neoclassical polarization embodied in $\mathcal{K}(t \to \infty)$.
  Comes down to calculating orbit averages in G, and then evaluating corresponding Φ.