Theory and modeling of alpha-driven TAEs in high-magnetic-field devices

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Outline

- Alpha particle physics of high field tokamaks
  - Introduction to high field tokamaks
  - Introduction to TAEs
- TAEs in high field tokamaks
  - Reduction of instability at same performance
- Determinants of TAE transport
  - Development of drift kinetic theory of transport
  - TAE amplitude that causes alpha depletion

Alpha particle physics of high field devices

Magnetic field and size determine DT fusion power P_f

- The goal of fusion research is a device that makes significant fusion power P_f

- In DT devices, fusion power is determined by pressure and volume:

$$P_f \sim \langle nT \rangle^2 V \sim \beta^2 B_0^4 V$$

- β^2 is determined by plasma physics

- B_0 and V are selected by device designers

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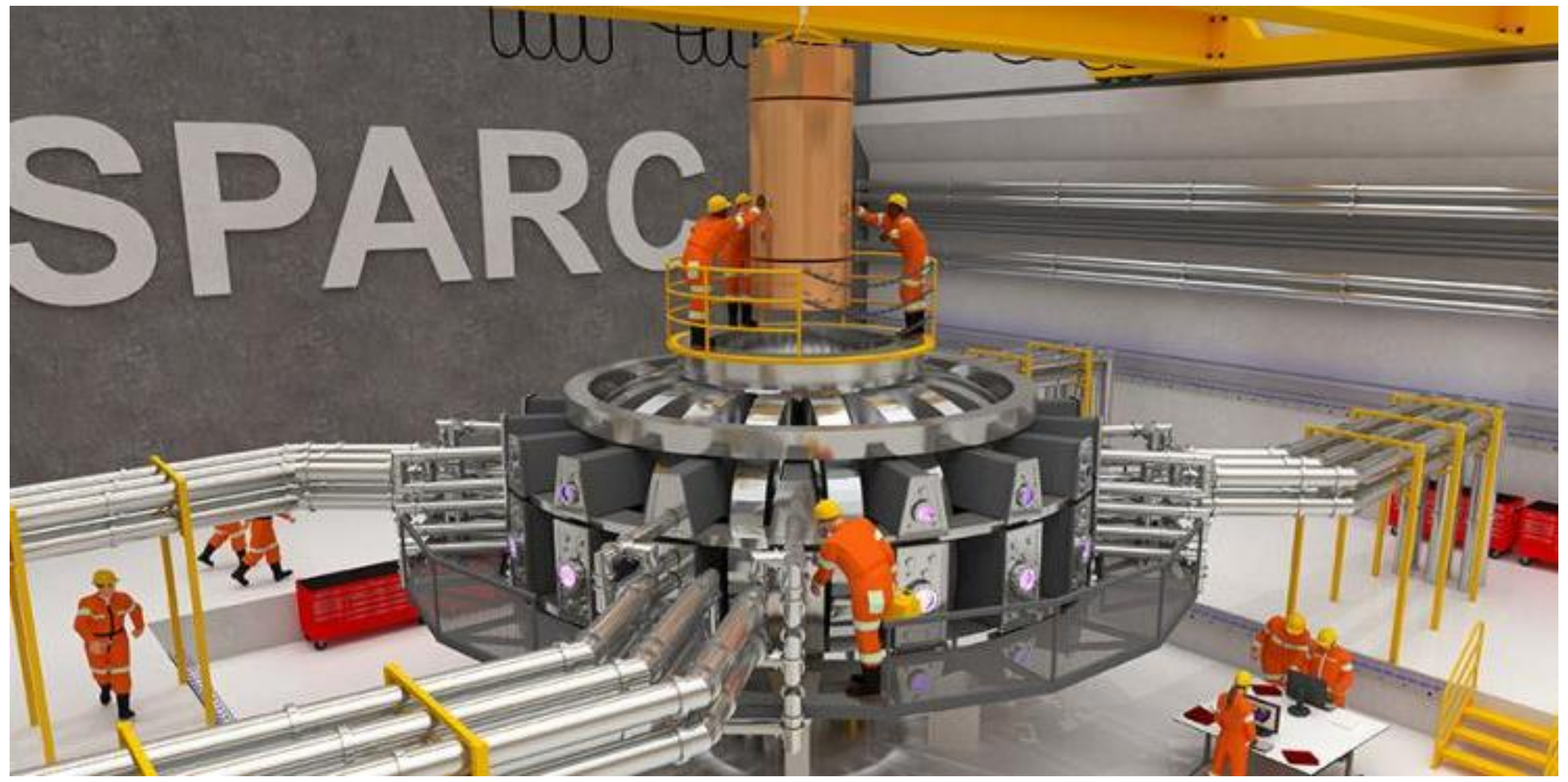
SPARC to ARC pathway exploits B_0 dependence of P_f

$$P_f \sim \langle nT \rangle^2 V \sim \beta^2 B_0^4 V$$

- Historically, magnet technology limited B_0
  - Large, expensive devices necessary
- New HTS technology allows higher B_0 and smaller, faster, cheaper devices
- SPARC is a small, high field HTS device with significant fusion power
- Expected DT operation ~2025
- High field DT reactor (ARC) will follow SPARC

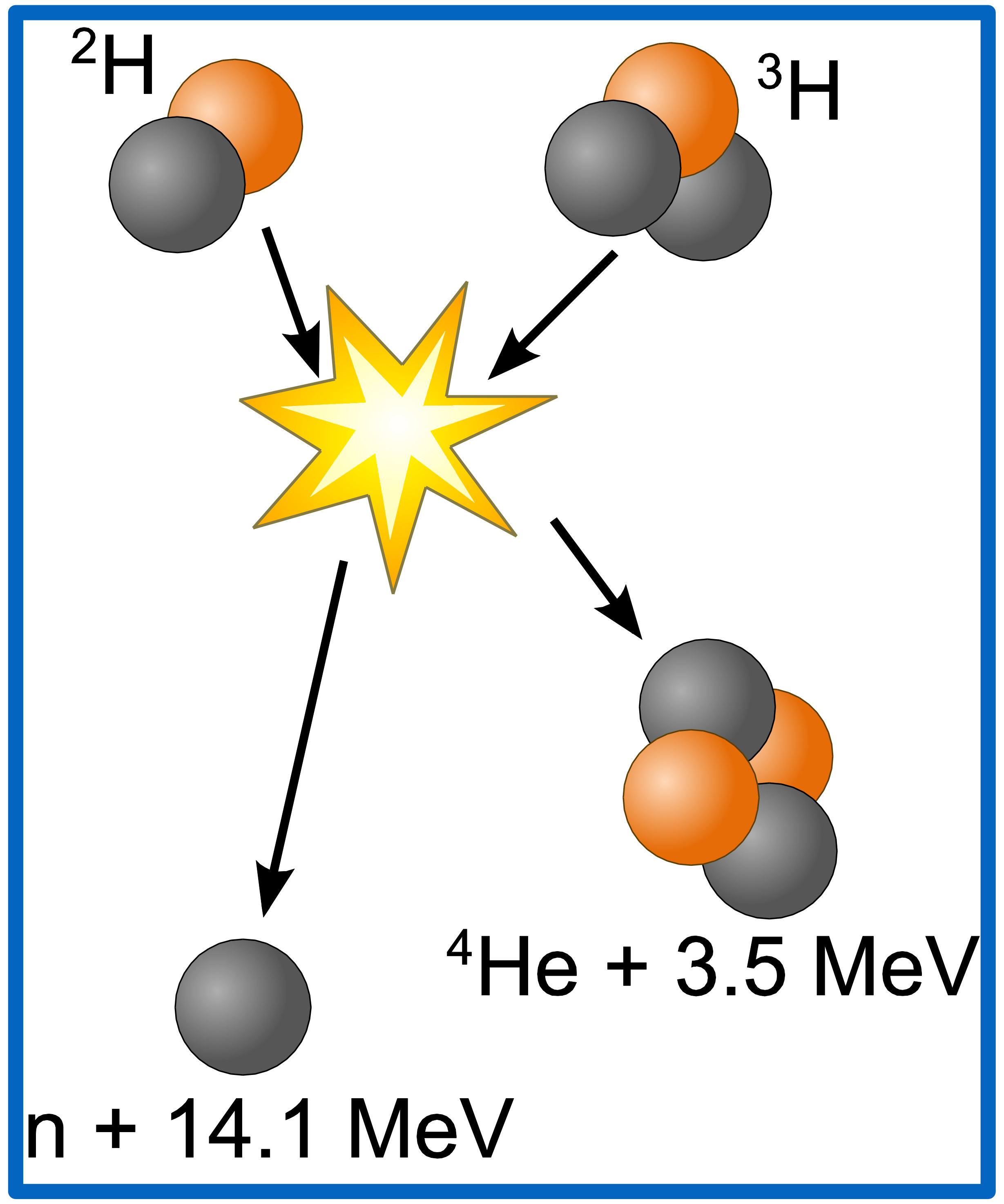
Quantity	ITER	SPARC V0
R_0	$6.2\ m$	$1.65\ m$
B_0	~ 5.3 T	~ 12 T
P_f	500 MW	50-100 MW



Alpha physics is novel part of next-generation DT tokamaks

- SPARC will be among first to produce large "amount" of alpha particles

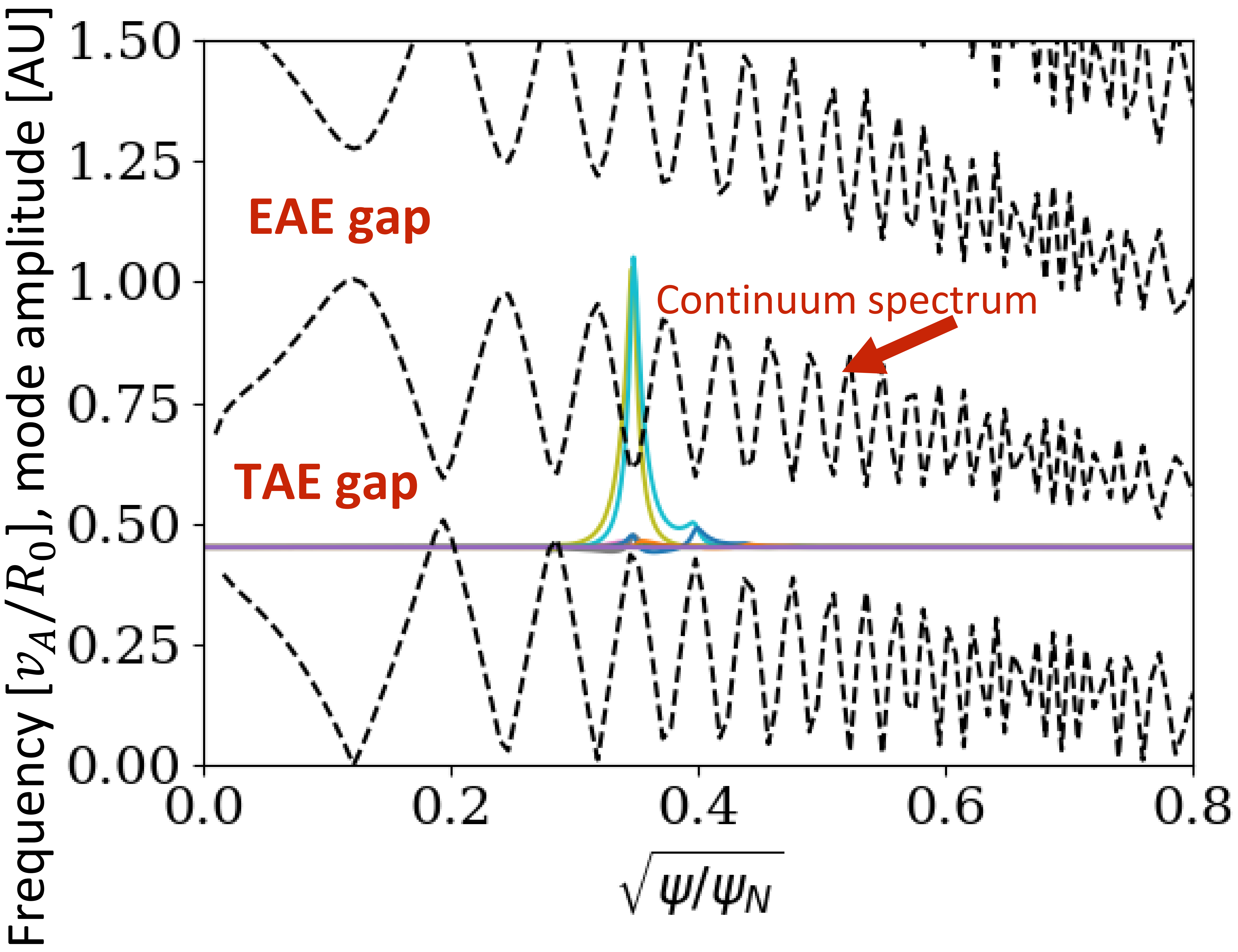
$n_e = 2n_D = 2n_T$	$T_i \sim T_e$	n_α	"T_α" [Calculated using average value of v^2 across slowing down distribution]	$n_\alpha T_\alpha/(2n_e T_e)$
$\sim 5 \times 10^{14}\ cm^{-3}$	$\sim 20\ keV$	$\sim 10^{12}\ cm^{-3}$	$\sim 580\ keV$	$\sim 15\%$



- Alpha confinement and transport is important
  - Transport can modify heat deposition and bulk plasma profiles
  - Alpha loss can degrade performance and damage device
- Alpha physics includes alpha-AE interaction

AEs are excited by alphas, can cause energetic particle loss

- AEs are shear Alfvén waves that exist as discrete modes
  - MHD structure determined by bulk
- Energetic particles, including alphas, destabilize AEs
  - Can be treated drift kinetically for wider modes
- AEs can transport alphas, degrading performance and damaging device
- ITER scenarios are unstable to AEs (see e.g. Rodrigues et al. NF 2015)



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Accelerated path to fusion poses urgent alpha physics questions

- Forthcoming high field DT experiments pose urgent theory and modeling questions:

  - Could high field devices have TAE activity?
    - Can it be reduced at same performance?
  - What amplitude TAE changes alpha population?
    - Development of transport theory required
    - Can the effect of this transport be reduced?

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TAEs in high field tokamaks

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Thermal species and energetic particles determine growth rate

- Consider a TAE of frequency $\omega = \frac{v_A}{2qR}$ driven by alphas born at speed v_0
- TAE linear growth rate is determined by sum of drive and damping
- Alpha drive is given by:

$$\gamma_\alpha \sim \omega q^2 \beta_\alpha F\left(\frac{v_A}{v_0}, \frac{1}{p_\alpha}\frac{dp_\alpha}{d\psi}\right)$$

Function of spatial gradient of alphas at resonant velocities

- Ion damping from species *j* is given by:

$$\gamma_j \sim -\omega q^2 \beta_j G\left(\frac{v_A}{v_{thj}}\right)$$

Function of energy gradient of ions at resonant velocities

Temperature controls AE stability

- Alpha particle density scales like:

$$n_\alpha \sim \alpha \text{ source rate} \times \text{slowing down time} \sim n_D n_T \langle \sigma v \rangle \left(T_e^{3/2} / n_e \right) \sim n_e T_e^{3.5}$$

- Neglecting resonance positions, ratio of AE drive to damping scales like:

$$\frac{Drive}{Damping} \sim \frac{\beta_\alpha}{\beta_j} \sim \frac{\frac{n_\alpha T_\alpha}{B_0^2}}{\frac{n_e T_e}{B_0^2}} \sim T_e^{2.5}$$

- AE drive/damping does not have explicit dependence on B_0

- High field scenarios are under development
  - Early concepts for SPARC [Greenwald et al. APS DPP CPP 2019] and ARC [Sorbom et al. FED 2015] have comparable temperatures to ITER
  - High field devices could have significant AE activity

Higher field devices can access higher absolute density

- At a given B_0, a tokamak can operate with a wide range of plasma parameters
- Overall trends in accessible parameters can be understood by holding tokamak figures of merit constant

Tokamak figures of merit	
$q_\star = \frac{2\pi a^2 B_0}{\mu_0 R_0 I_p}\left(\frac{1+\kappa^2}{2}\right)$	$f_{GW} = \frac{\bar{n}_e}{I_p/\pi a^2}$

- Resulting trend in *accessible* core plasma parameters is:

Trend with $\boldsymbol{B_0}$	
$I_p \sim B_0$	$n_e \sim B_0$

- Current high field concepts operate far from maximum density

AEs can be reduced by operating at higher *n*, lower *T*

- Economically, reactor should achieve fixed power density,

$$p_f\left[\frac{MW}{m^3}\right] = E_f n_D n_T \langle \sigma v \rangle \sim \mathrm{n_e^2 T_e^2} \equiv C$$

- At a given power density,

$$\frac{Drive}{Damping} \sim T_e^{2.5} \sim \frac{C^{1.25}}{n_e^{2.5}}$$

- High field devices can access very high densities (higher than in low field machines)
- For fixed power density, this corresponds to lower temperatures
- This slows down alphas faster, reducing AE drive while maintaining fusion power density

High field devices can reduce AE instability by operating at higher density

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Determinants of TAE transport

Unperturbed alphas described by slowing down distribution

- Unperturbed alpha population given by balancing fusion source against drag:

$$C_{drag}\{f_0\} + \frac{S_{fus}\,\delta(v-v_0)}{4\,\pi v^2} = 0$$

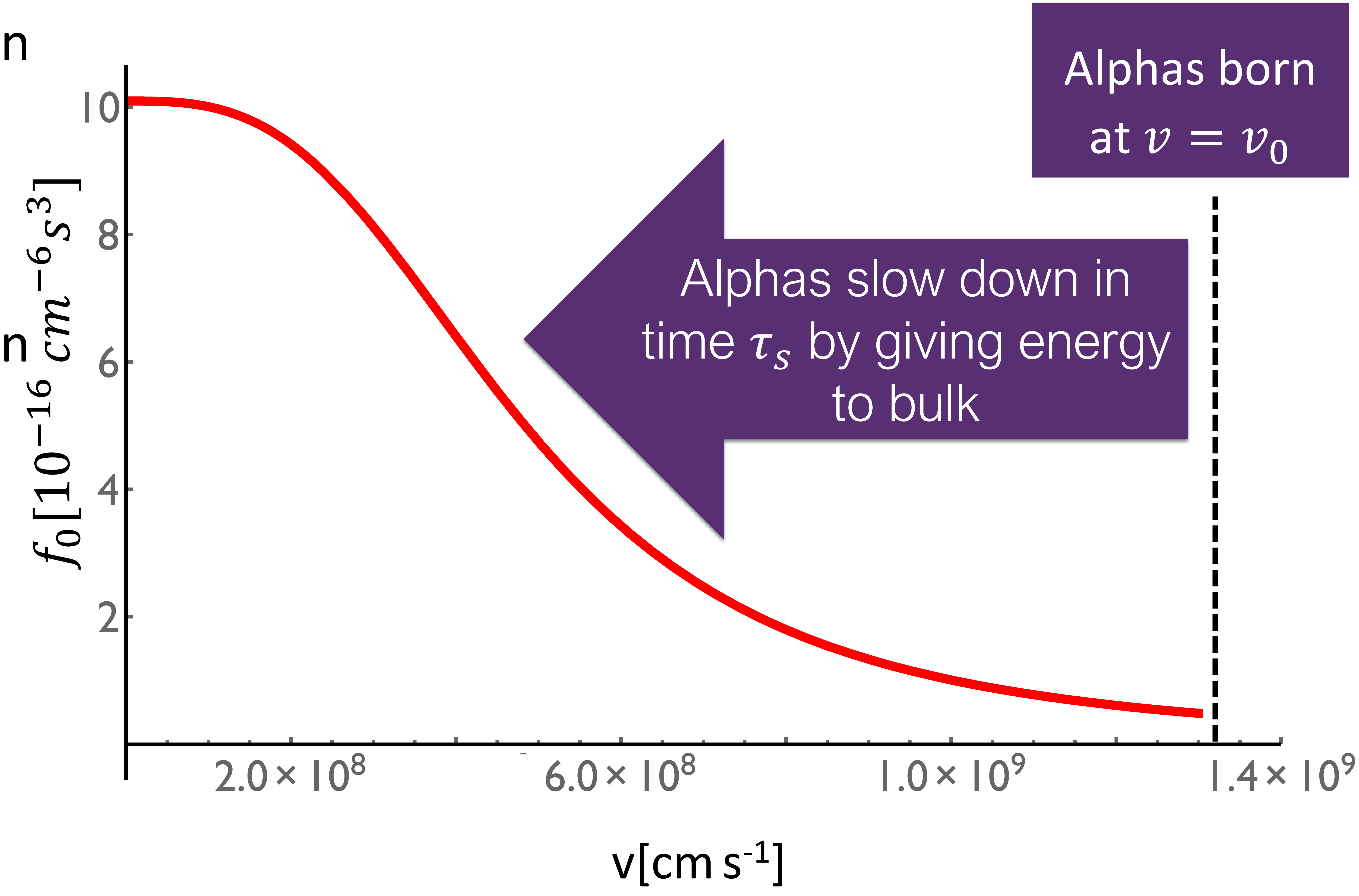
- Gives the slowing down distribution:

$$f_0 = \frac{S_{fus}\tau_s\,H(v-v_0)}{4\,\pi(v^3+v_c^3)}$$

$$C_{drag}\{f\} = \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}[(v^3+v_c^3)f]$$

$$v_c^3 = \frac{3\sqrt{\pi}T_e^{\frac{3}{2}}}{\sqrt{2m_e}n_e}\sum_i \frac{Z_i^2 n_i}{M_i},$$

$$\tau_s = \frac{3MT_e^{3/2}}{4(2\pi m_e)^{1/2}Z^2e^4n_e \ln\Lambda}$$



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Drift kinetic formulation aims to put limit on TAE amplitude

- TAEs could transport alphas with a diffusion coefficient D_{TAE}
- TAE transport significantly changes plasma when

$$\boxed{\frac{D_{TAE}\tau_s}{a_\alpha^2} \sim 1}$$

Alphas removed by TAE before they can heat the local background plasma

- We develop a drift kinetic theory for D_{TAE} to determine mode amplitude where $\frac{D_{TAE}\tau_s}{a_\alpha^2} \sim 1$
- Focus on collisional processes (not stochasticity arising from resonance overlap)

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TAEs perturb tokamak magnetic field

- Background magnetic field is of standard form: $\vec{B} = \nabla\alpha \times \nabla\psi$, $\alpha = \zeta - q\vartheta$

- AEs include magnetic field and electric field

  Perturbed vector potential:

$$\overrightarrow{A_1} = A_{\parallel} e^{i(n\,\alpha - \omega t + (nq-m)\vartheta)} e^{i\int d\psi k_\psi(\psi)} \hat{b}$$

Amplitude — Wave phase — Radial variation

  Perturbed electric potential:

$$\Phi_1 = \Phi e^{i(n\,\alpha - \omega t + (nq-m)\vartheta)} e^{i\int d\psi k_\psi(\psi)}$$

Electric field: $\overrightarrow{E_1} = -\nabla\Phi_1 - \frac{1}{c}\frac{\partial \overrightarrow{A_1}}{\partial t}$

Magnetic field: $\overrightarrow{B_1} = \nabla \times \overrightarrow{A_1}$

- Modes create a corresponding perturbation to f_0

$$f_1 = \frac{Ze\,\Phi_1}{M}\frac{\partial f_0}{\partial \mathcal{E}} + h e^{i(n\,\alpha - \omega t + (nq-m)\vartheta)} e^{i\int d\psi k_\psi(\psi)}$$

Adiabatic response,
$\mathcal{E}$ is energy

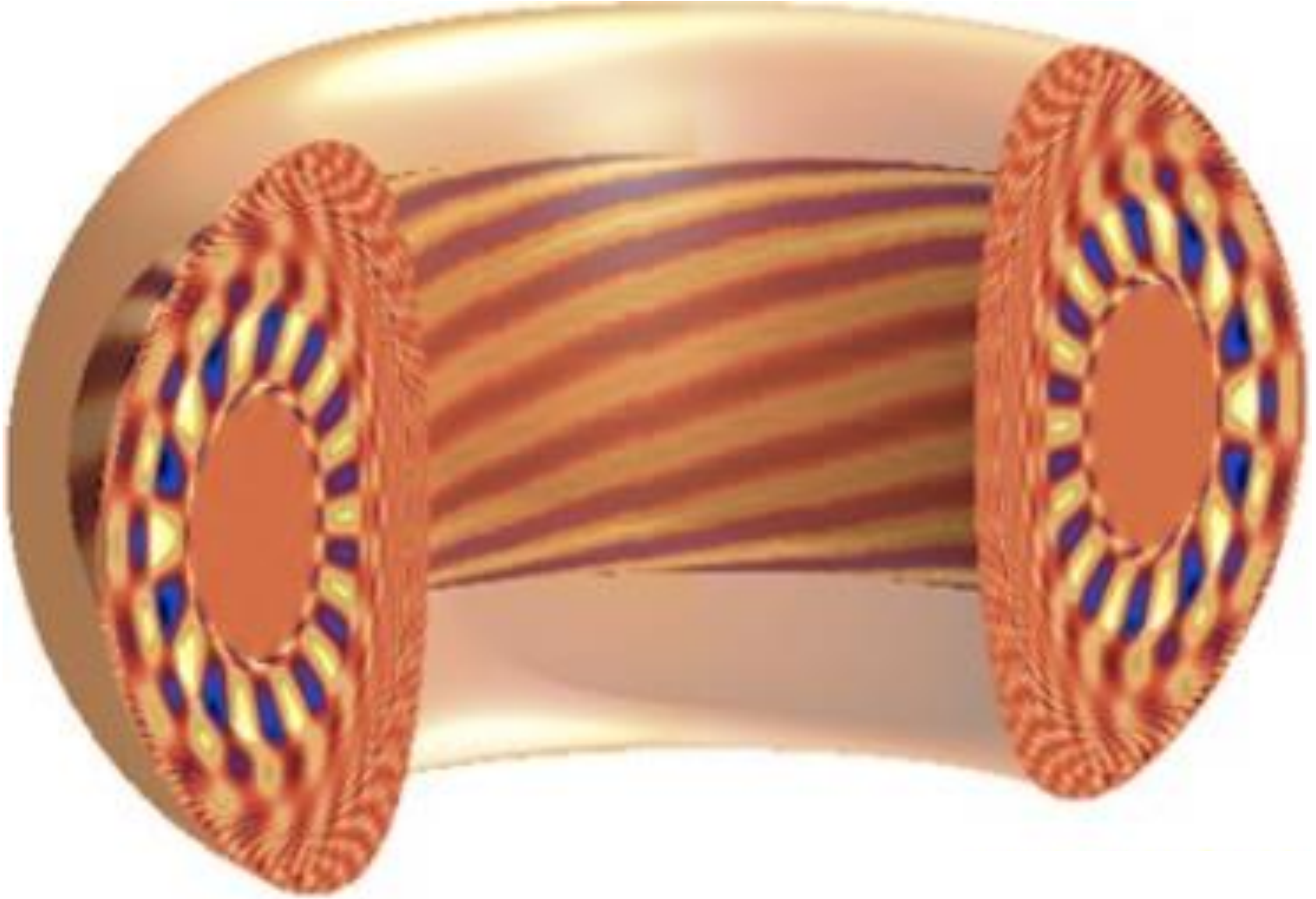

Image credit: Heidbrink APS 2007

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Perturbed drift kinetic equation models transport

- To first order, the drift kinetic equation reads

$$\partial_t f_1 + \left(v_\parallel \hat{b} + \overrightarrow{v_d}\right) \cdot \nabla f_1 + \left[\left(v_\parallel \hat{b}_1 + \overrightarrow{v_{d1}}\right) \cdot \nabla\psi\right] \frac{\partial f_0}{\partial\psi} + \left[\frac{Ze}{M}\left(v_\parallel \hat{b} + \overrightarrow{v_d}\right) \cdot \overrightarrow{E_1}\right] \frac{\partial f_0}{\partial\mathcal{E}} = C\{f_1\}$$

- For AE modes, this reads:

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial\vartheta} - i\left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha - k_\psi \overrightarrow{v_d} \cdot \nabla\psi\right)h - C\{f_1\} = -cn\left(1 - \frac{\omega}{\omega_\star}\right)\left(\Phi - \frac{v_\parallel}{c} A_\parallel\right)\frac{\partial f_0}{\partial\psi} e^{i(nq-m)\vartheta}$$

- Here, we define $\omega_\star \equiv \dfrac{cMn\frac{\partial f_0}{\partial\psi}}{Ze\frac{\partial f_0}{\partial\mathcal{E}}}$

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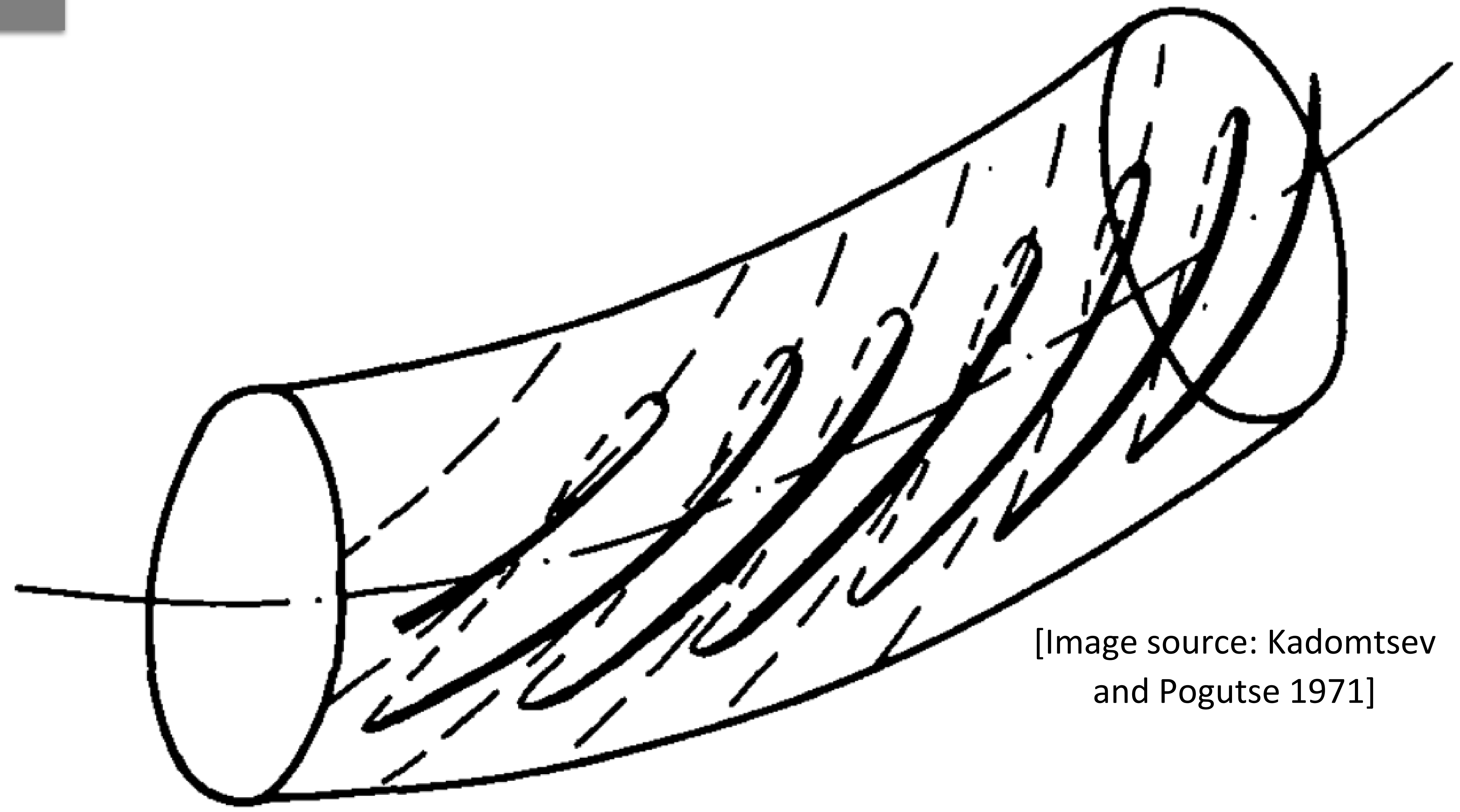
Perturbed drift kinetic equation models transport

$$v_\parallel \hat{b} \cdot \nabla \vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\vec{v_d} \cdot \nabla \alpha - k_\psi \vec{v_d} \cdot \nabla \psi\right) h - C\{f_1\} = -cn\left(1 - \frac{\omega}{\omega_\star}\right)\left(\Phi - \frac{v_\parallel}{c} A_\parallel\right)\frac{\partial f_0}{\partial \psi} e^{i(nq-m)\vartheta}$$

Streaming along magnetic field



[Image source: Kadomtsev and Pogutse 1971]

Perturbed drift kinetic equation models transport

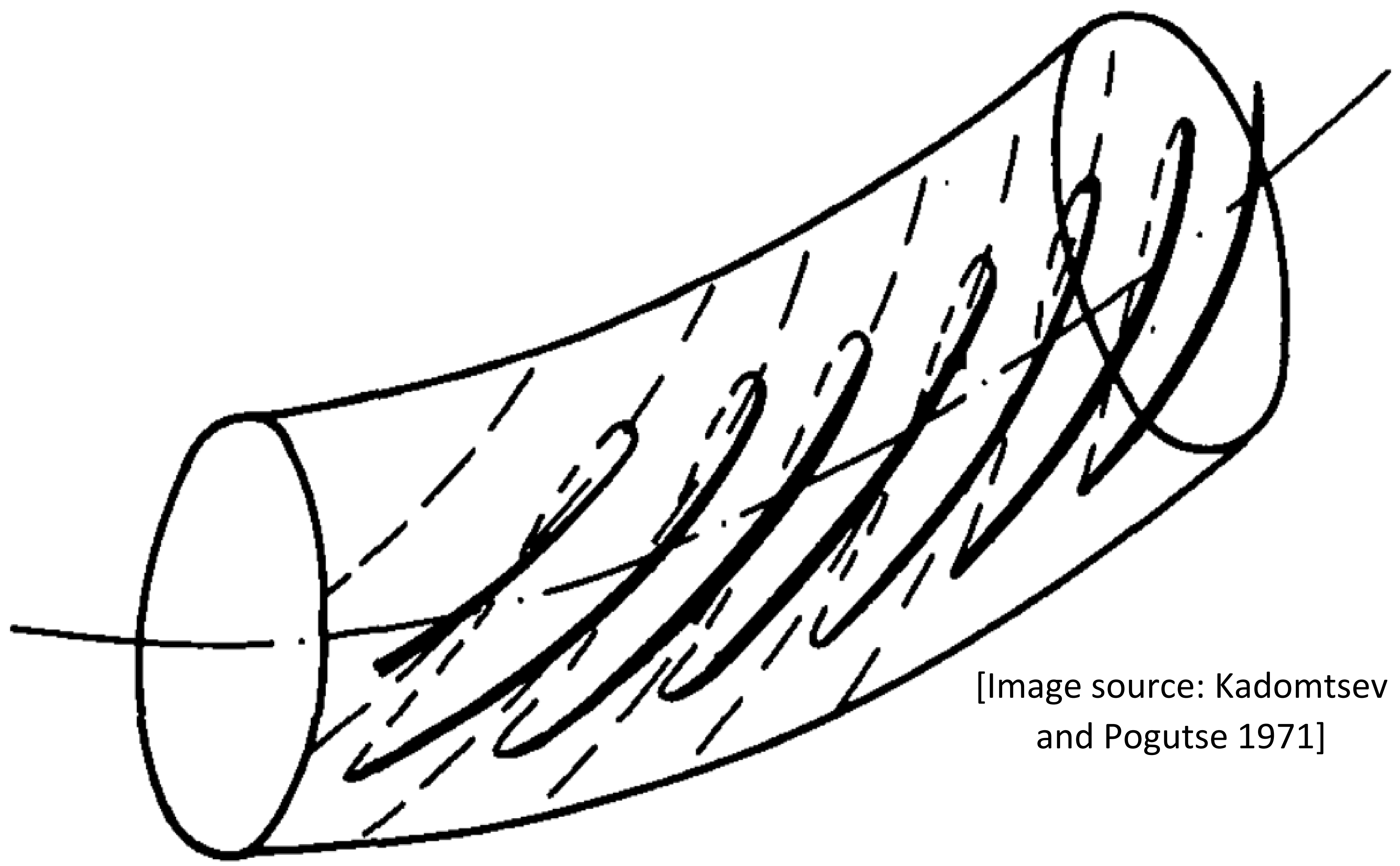
$$v_{\|}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta}-i\left(\omega-n\overrightarrow{v_d}\cdot\nabla\alpha-k_\psi\overrightarrow{v_d}\cdot\nabla\psi\right)h-C\{f_1\}=-cn\left(1-\frac{\omega}{\omega_\star}\right)\left(\Phi-\frac{v_{\|}}{c}A_{\|}\right)\frac{\partial f_0}{\partial\psi}e^{i(nq-m)\vartheta}$$

Wave frequency

Perturbed drift kinetic equation models transport

$$v_{\parallel}\hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha - k_{\psi}\overrightarrow{v_d} \cdot \nabla\psi\right)h - C\{f_1\} = -cn\left(1 - \frac{\omega}{\omega_{\star}}\right)\left(\Phi - \frac{v_{\parallel}}{c}A_{\parallel}\right)\frac{\partial f_0}{\partial \psi} e^{i(nq-m)\vartheta}$$

Tangential precession

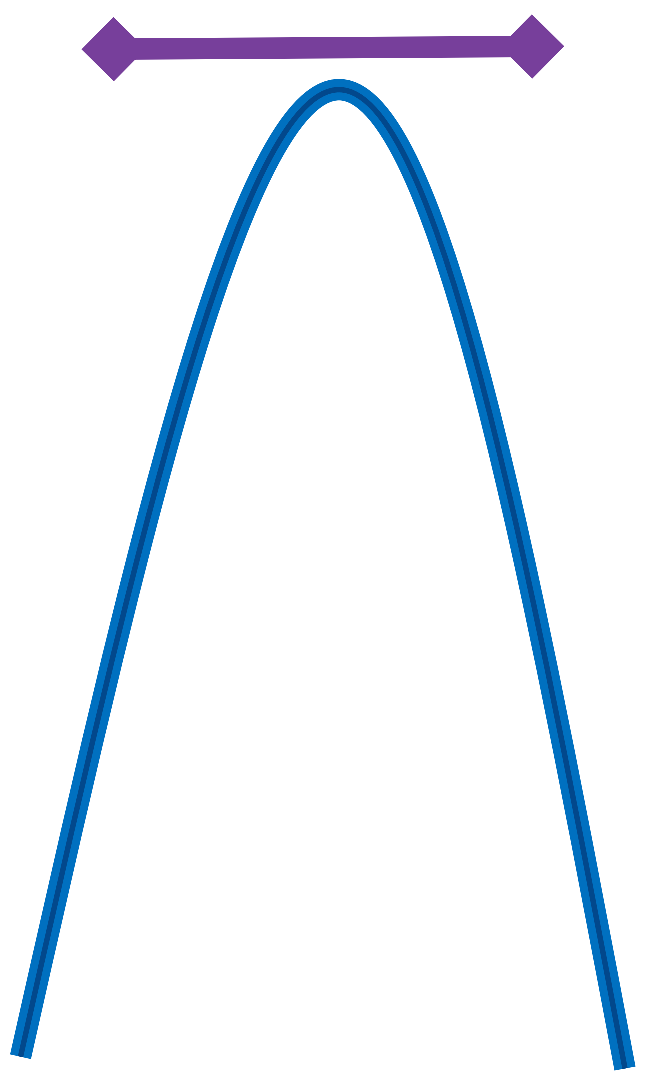


[Image source: Kadomtsev and Pogutse 1971]

Perturbed drift kinetic equation models transport

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha - k_\psi \overrightarrow{v_d} \cdot \nabla\psi\right)h - C\{f_1\} = -cn\left(1 - \frac{\omega}{\omega_\star}\right)\left(\Phi - \frac{v_\parallel}{c}A_\parallel\right)\frac{\partial f_0}{\partial \psi} e^{i(nq-m)\vartheta}$$

Radial drift off flux surface



Particle orbit width

Mode amplitude as function of ψ

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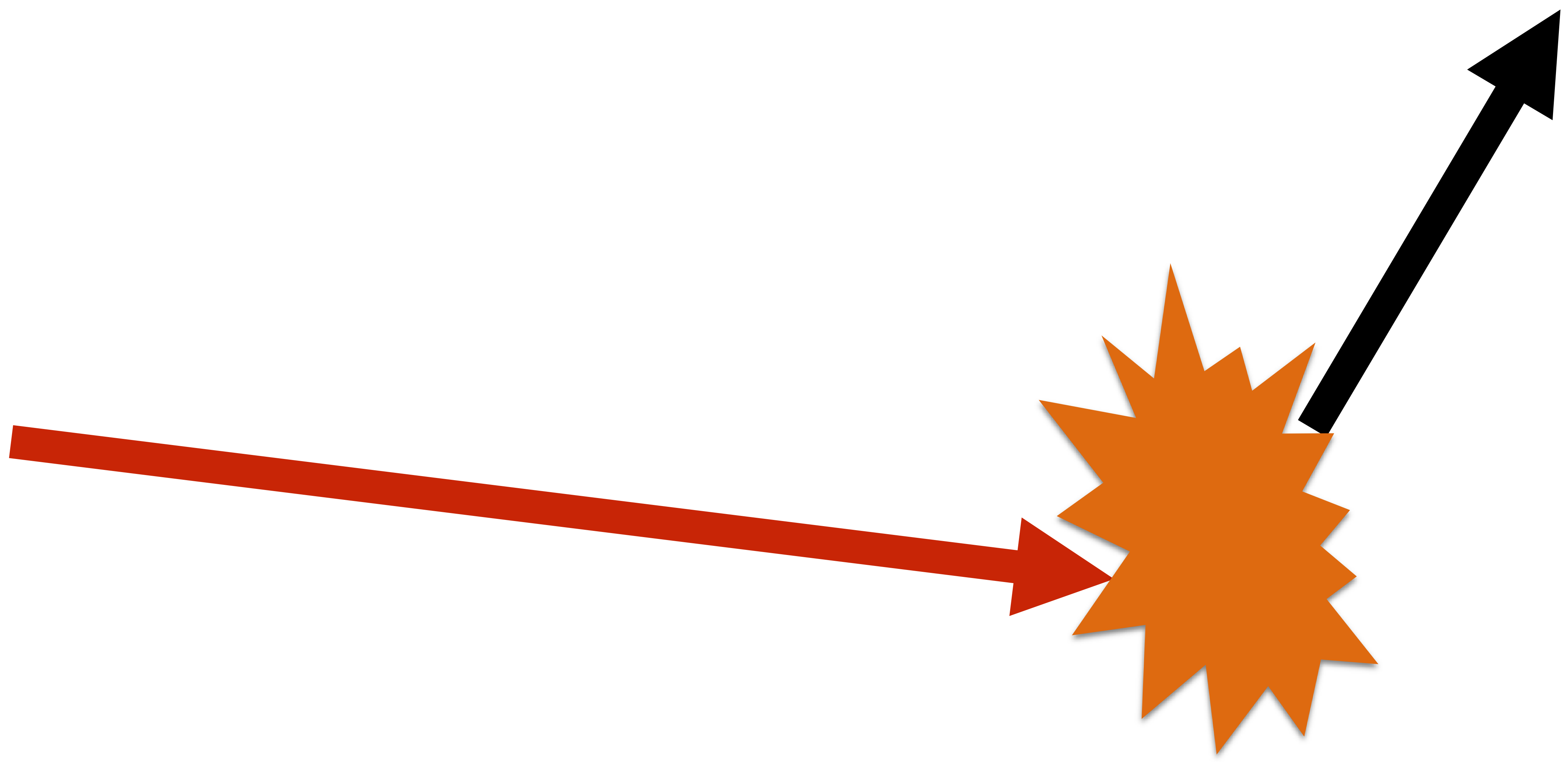
Perturbed drift kinetic equation models transport

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha - k_\psi \overrightarrow{v_d} \cdot \nabla\psi\right)h - C\{f_1\} = -cn\left(1 - \frac{\omega}{\omega_\star}\right)\left(\Phi - \frac{v_\parallel}{c} A_\parallel\right)\frac{\partial f_0}{\partial \psi} e^{i(nq-m)\vartheta}$$

Pitch angle scattering



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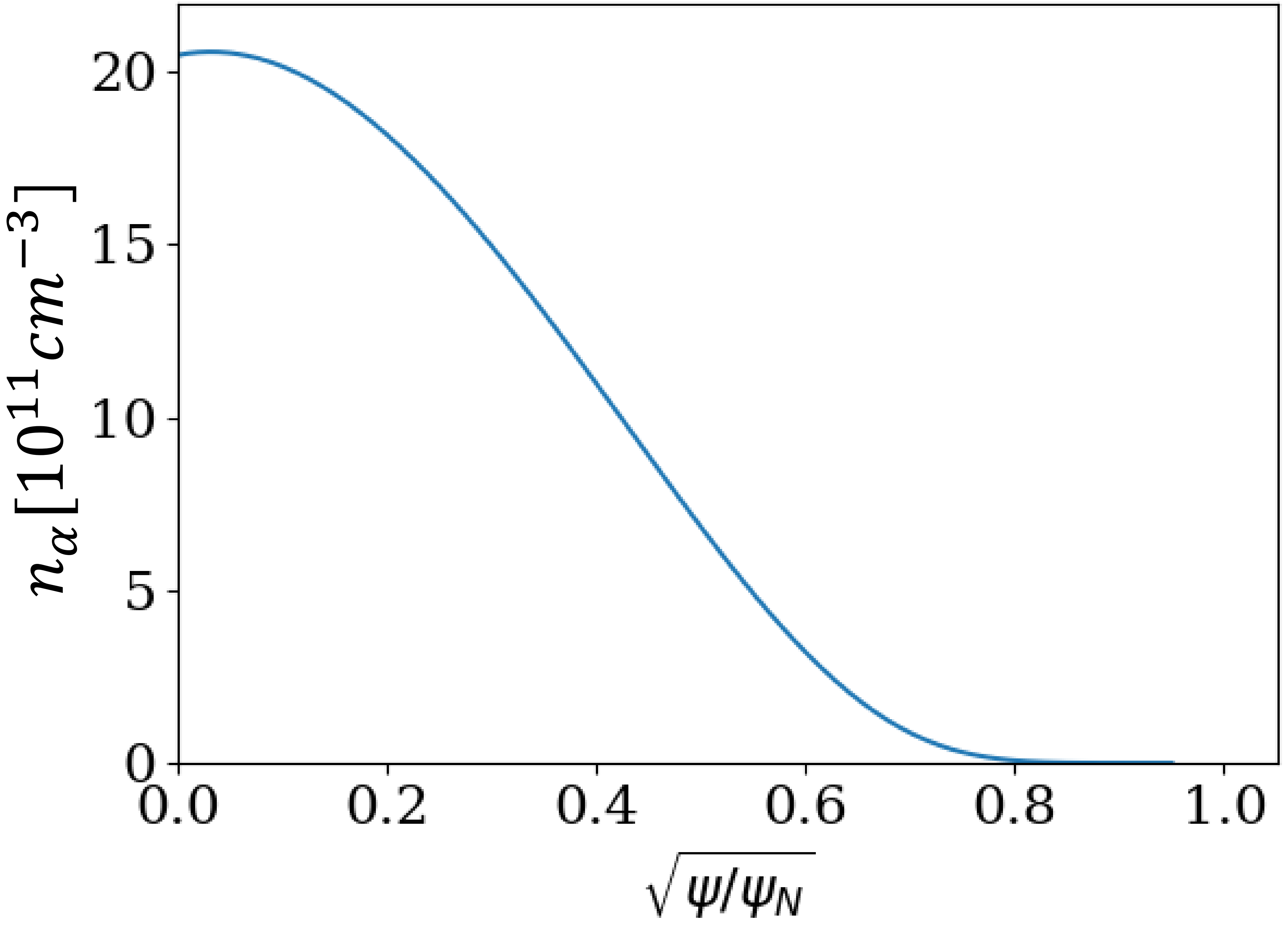
Perturbed drift kinetic equation models transport

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\vec{v_d}\cdot\nabla\alpha - k_\psi \vec{v_d}\cdot\nabla\psi\right)h - C\{f_1\} = -cn\left(1 - \frac{\omega}{\omega_\star}\right)\left(\Phi - \frac{v_\parallel}{c}A_\parallel\right)\frac{\partial f_0}{\partial \psi} e^{i(nq-m)\vartheta}$$

Drive due to perturbation and background spatial gradient of alphas



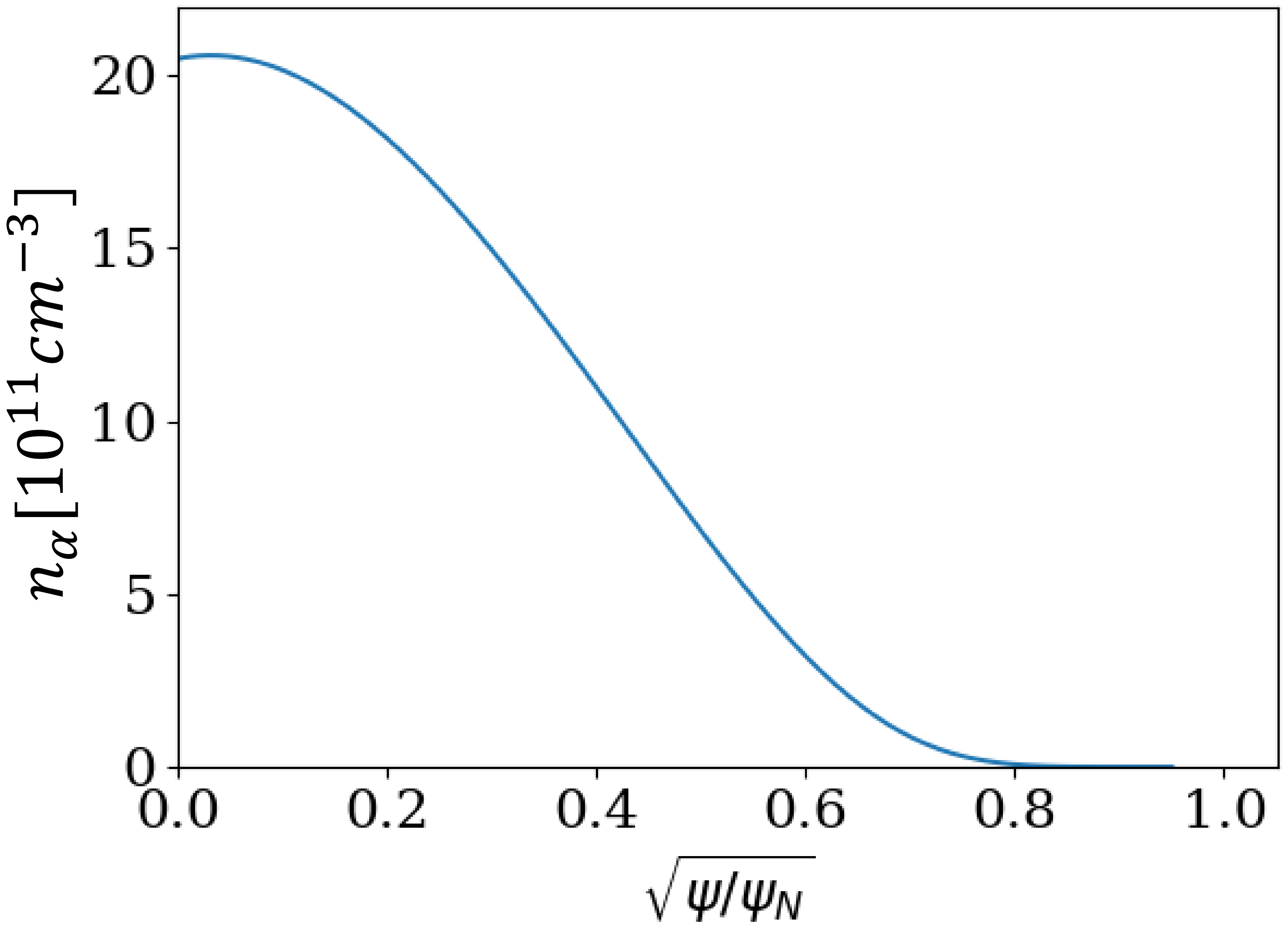
Note: $\omega_\star \equiv \frac{cMn\frac{\partial f_0}{\partial \psi}}{Ze\frac{\partial f_0}{\partial \mathcal{E}}} > \omega$ for instability

From now on,

$$cn\left(1 - \frac{\omega}{\omega_\star}\right)\left(\Phi - \frac{v_\parallel}{c}A_\parallel\right)\frac{\partial f_0}{\partial \psi} \equiv S(\vartheta)$$

Perturbed drift kinetic equation models transport

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\vec{v_d}\cdot\nabla\alpha - k_\psi \vec{v_d}\cdot\nabla\psi\right)h - C\{f_1\} = -S(\vartheta)e^{i(nq-m)\vartheta}$$



Drive due to perturbation and background spatial gradient of alphas

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Transport occurs via pitch angle scatter of resonant alphas

$$v_{\|}\hat{b}\cdot\nabla\vartheta\,\frac{\partial h}{\partial\vartheta}-i\left(\omega-n\overrightarrow{v_d}\cdot\nabla\alpha-k_\psi\overrightarrow{v_d}\cdot\nabla\psi\right)h-C\{f_1\}=-S(\vartheta)e^{i(nq-m)\vartheta}$$

Particle in phase with wave

Particle moved radially by wave

Particle decorrelates via pitch angle scatter: $C_{pas}\{h\}\sim\nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}$

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Particle in phase with wave

Particle moved radially by wave

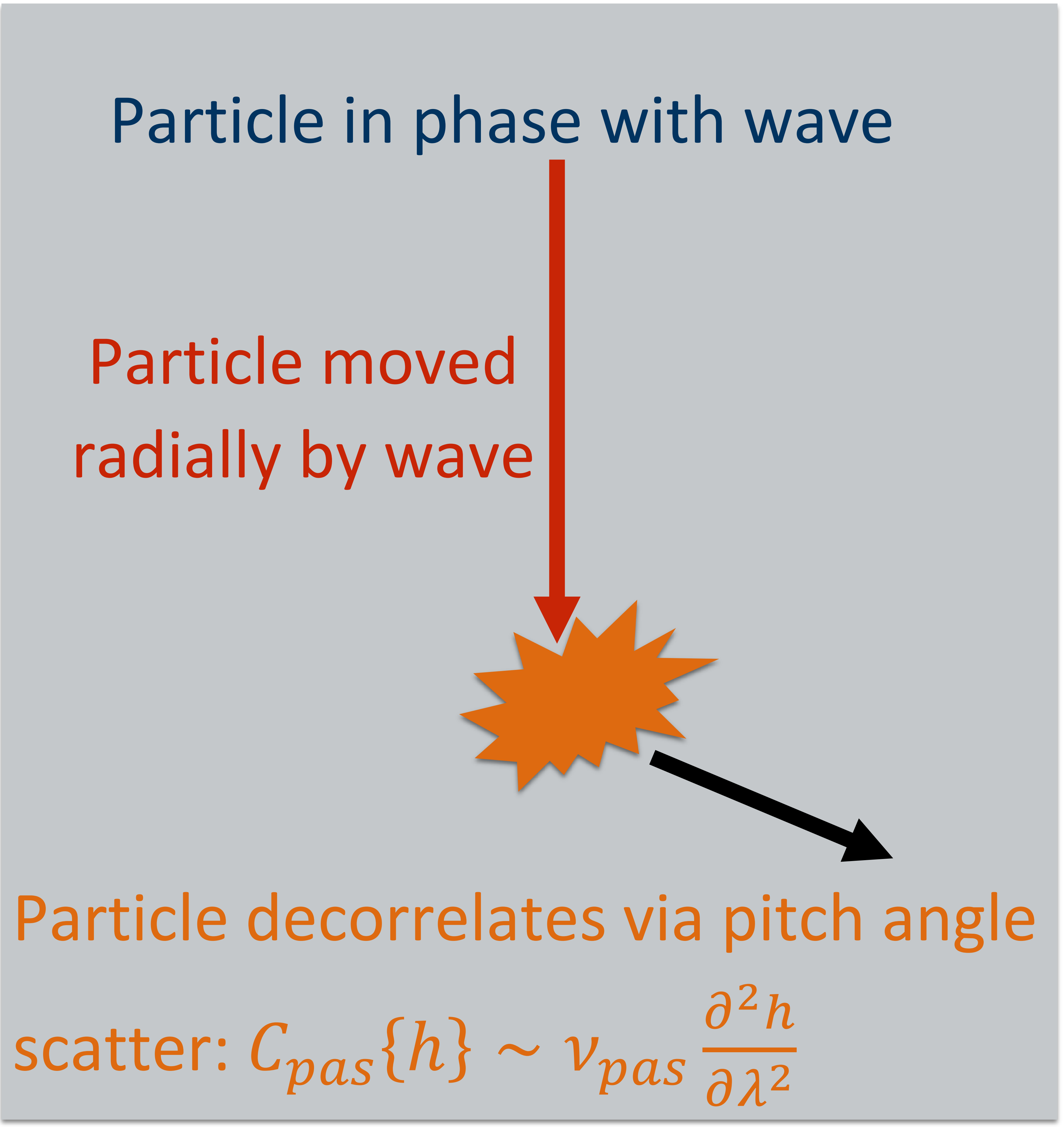
Particle decorrelates via pitch angle scatter: $C_{pas}\{h\}\sim\nu_{pas}\frac{\partial^2 h}{\partial\lambda^2}$

- Fraction of particles in phase: $\omega-\vec{k}\cdot\overrightarrow{v_d}\sim\omega\delta\lambda\sim\frac{\nu_{pas}}{\delta\lambda^2}\rightarrow\delta\lambda\sim\left(\frac{\nu_{pas}}{\omega}\right)^{1/3}$

Higher v_{pas} allows more particles to interact with wave

Transport occurs via pitch angle scatter of resonant alphas

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha - k_\psi \overrightarrow{v_d} \cdot \nabla\psi\right)h - C\{f_1\} = -S(\vartheta)e^{i(nq-m)\vartheta}$$



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  Higher v_{pas} allows more particles to interact with wave

- Decorrelation time: $\delta t \sim \frac{\delta\lambda^2}{\nu_{pas}} \sim \frac{1}{\omega^{2/3}\nu_{pas}^{1/3}}$

  Higher v_{pas} creates faster decorrelation

- Radial step: $v_\psi \delta t \sim \frac{v_\psi}{\omega^{2/3}\nu_{pas}^{1/3}}$

  Faster decorrelation time shortens step size

Transport occurs via pitch angle scatter of resonant alphas

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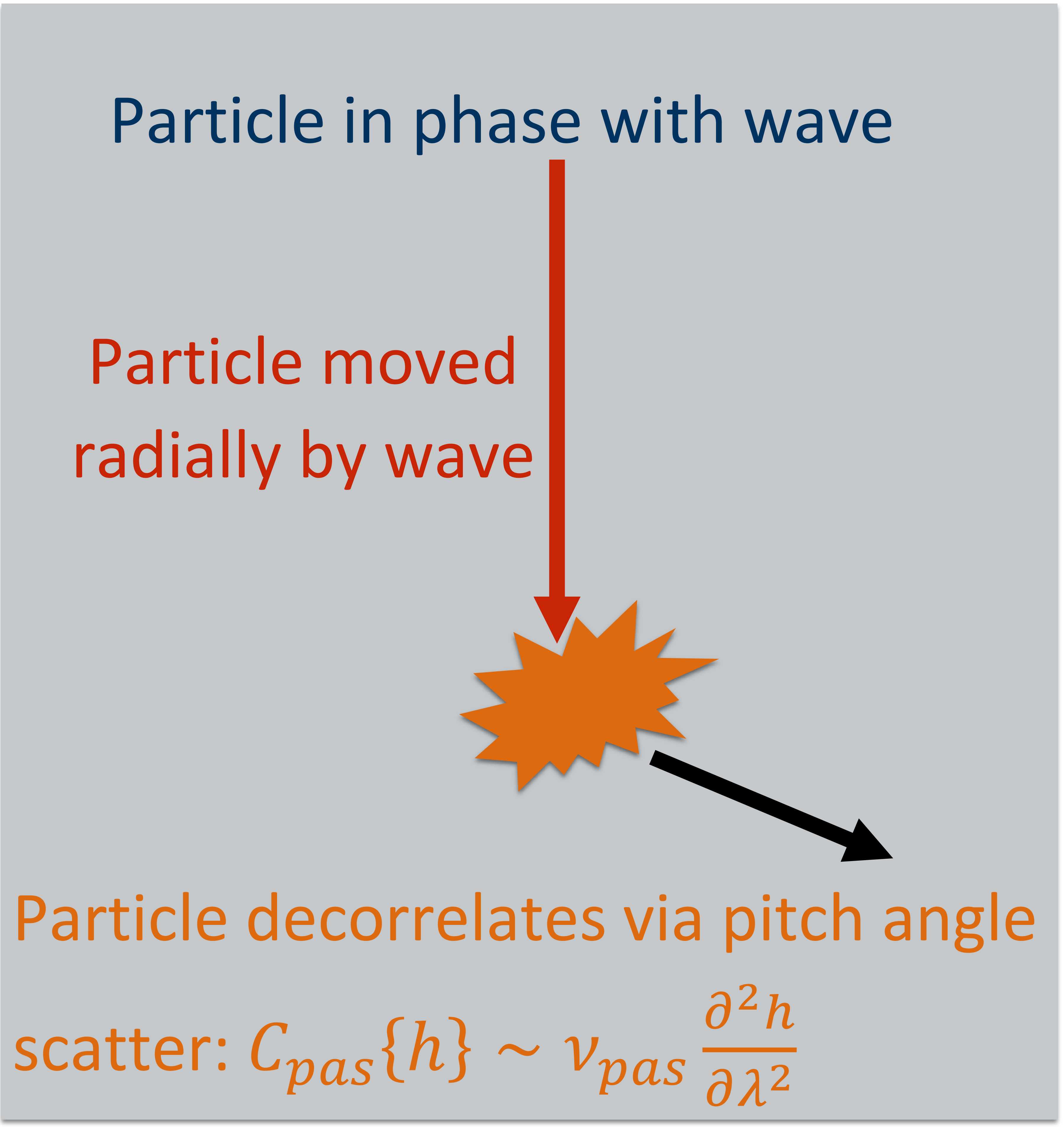

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  Faster decorrelation time shortens step size

- Overall flux $\Gamma\sim-\delta\lambda\frac{(v_\psi\delta t)^2}{\delta t}\frac{\partial n_\alpha}{\partial\psi}$: $\boxed{\Gamma\sim-\frac{v_\psi^2}{\omega}\frac{\partial n_\alpha}{\partial\psi}}$

v_{pas} dependence of decorrelation time and fraction of particles cancel out: use Krook operator

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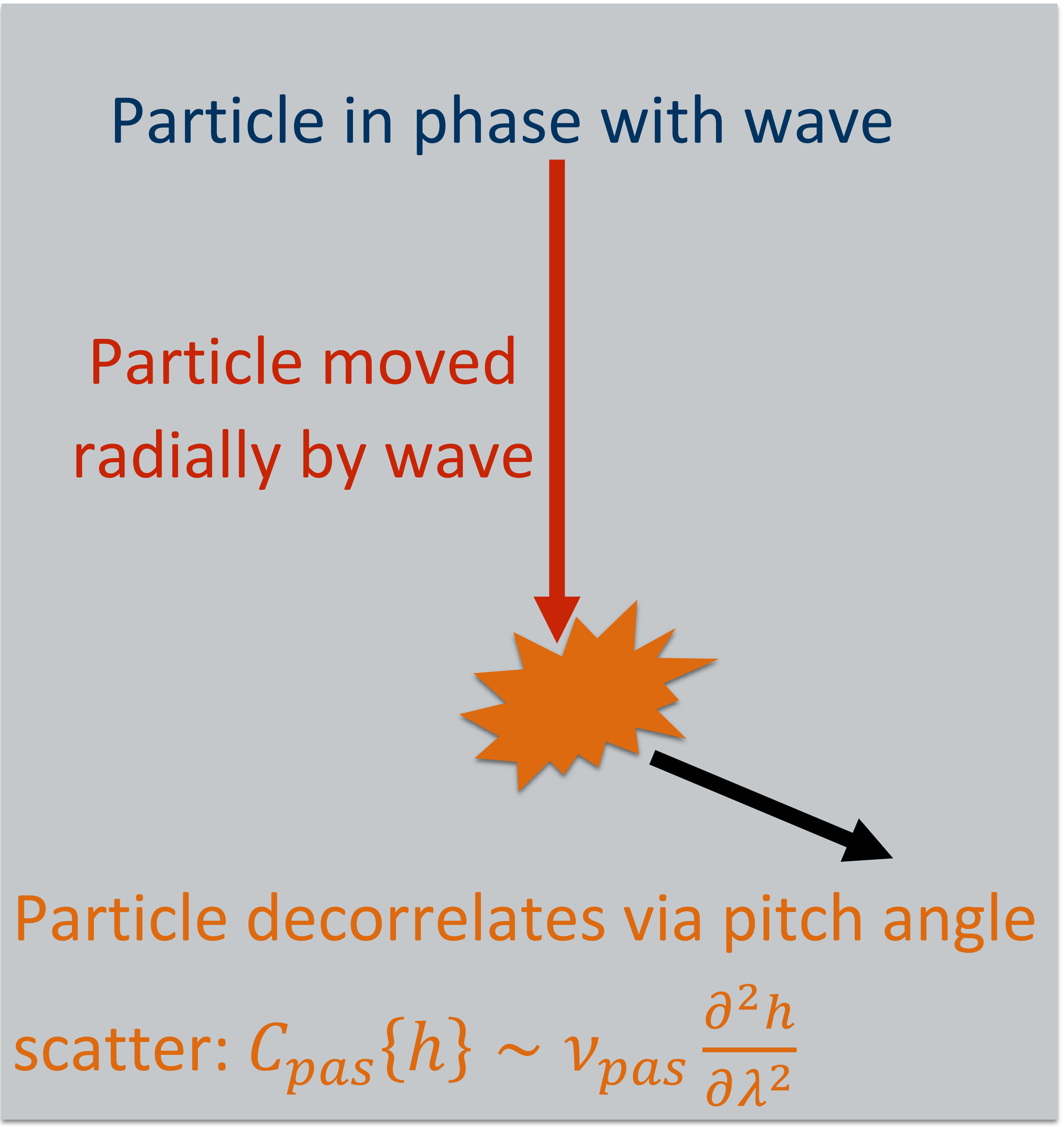
Transport occurs via pitch angle scatter of resonant alphas

$$v_\parallel \hat{b} \cdot \nabla\vartheta \frac{\partial h}{\partial \vartheta} - i\left(\omega - n\overrightarrow{v_d}\cdot\nabla\alpha - k_\psi \overrightarrow{v_d}\cdot\nabla\psi\right)h + \nu h = -S(\vartheta) e^{i(nq-m)\vartheta}$$



- Fraction of particles in phase: $\omega - \vec{k}\cdot\overrightarrow{v_d} \sim \omega\delta\lambda \sim \frac{\nu_{pas}}{\delta\lambda^2} \rightarrow \delta\lambda \sim \left(\frac{\nu_{pas}}{\omega}\right)^{1/3}$

Higher ν_{pas} allows more particles to interact with wave

- Decorrelation time: $\delta t \sim \frac{\delta\lambda^2}{\nu_{pas}} \sim \frac{1}{\omega^{2/3}\nu_{pas}^{1/3}}$

Higher ν_{pas} creates faster decorrelation

- Radial step: $v_\psi \delta t \sim \frac{v_\psi}{\omega^{2/3}\nu_{pas}^{1/3}}$

Faster decorrelation time shortens step size

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ν_{pas} dependence of decorrelation time and fraction of particles cancel out: use Krook operator

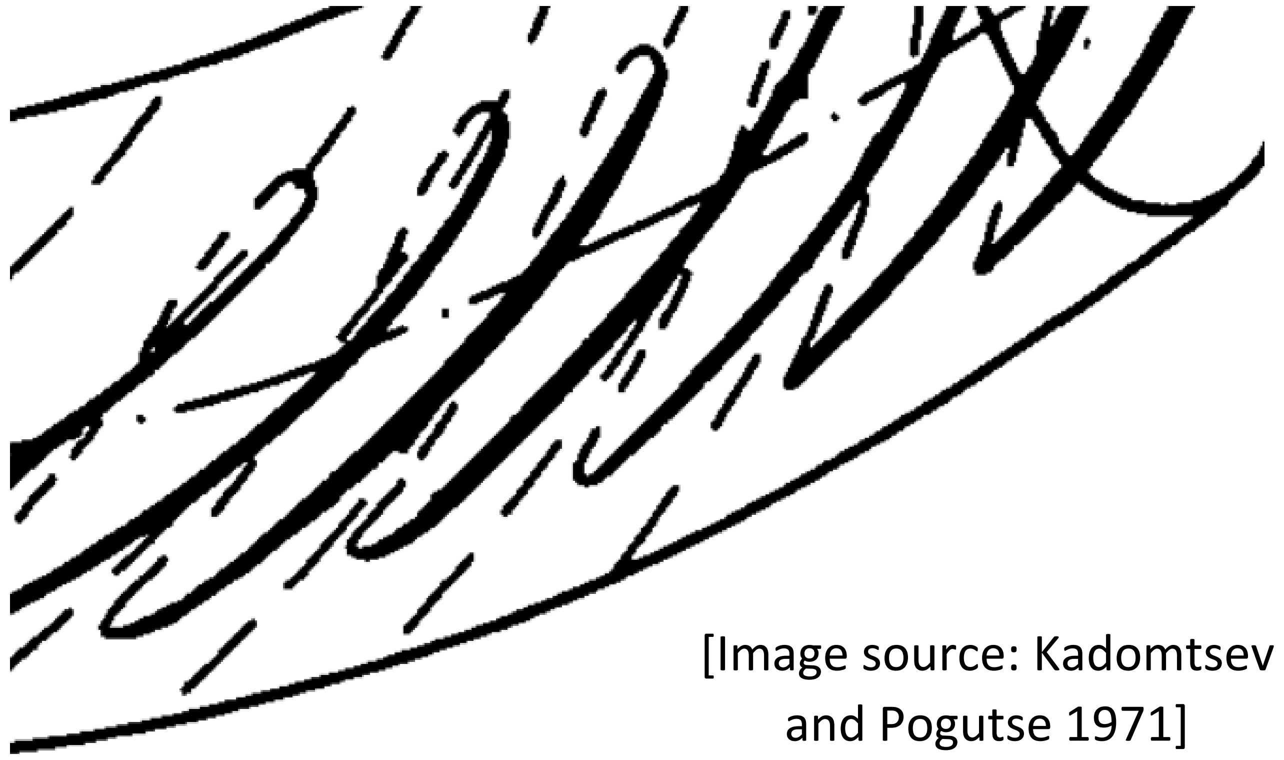
Integration over particle trajectory gives *h*

- Trapped periodicity means only integrate over full bounce (bounce time=τ_b) giving:

$$h = -\left(\oint S[\vartheta(\tau)]P(\tau)d\tau\right)\left(\frac{\nu\tau_b}{\left(\omega - n\overline{\vec{v}_d \cdot \nabla\alpha}\right)^2\tau_b^2 + \nu^2\tau_b^2}\right)$$

Phase factor

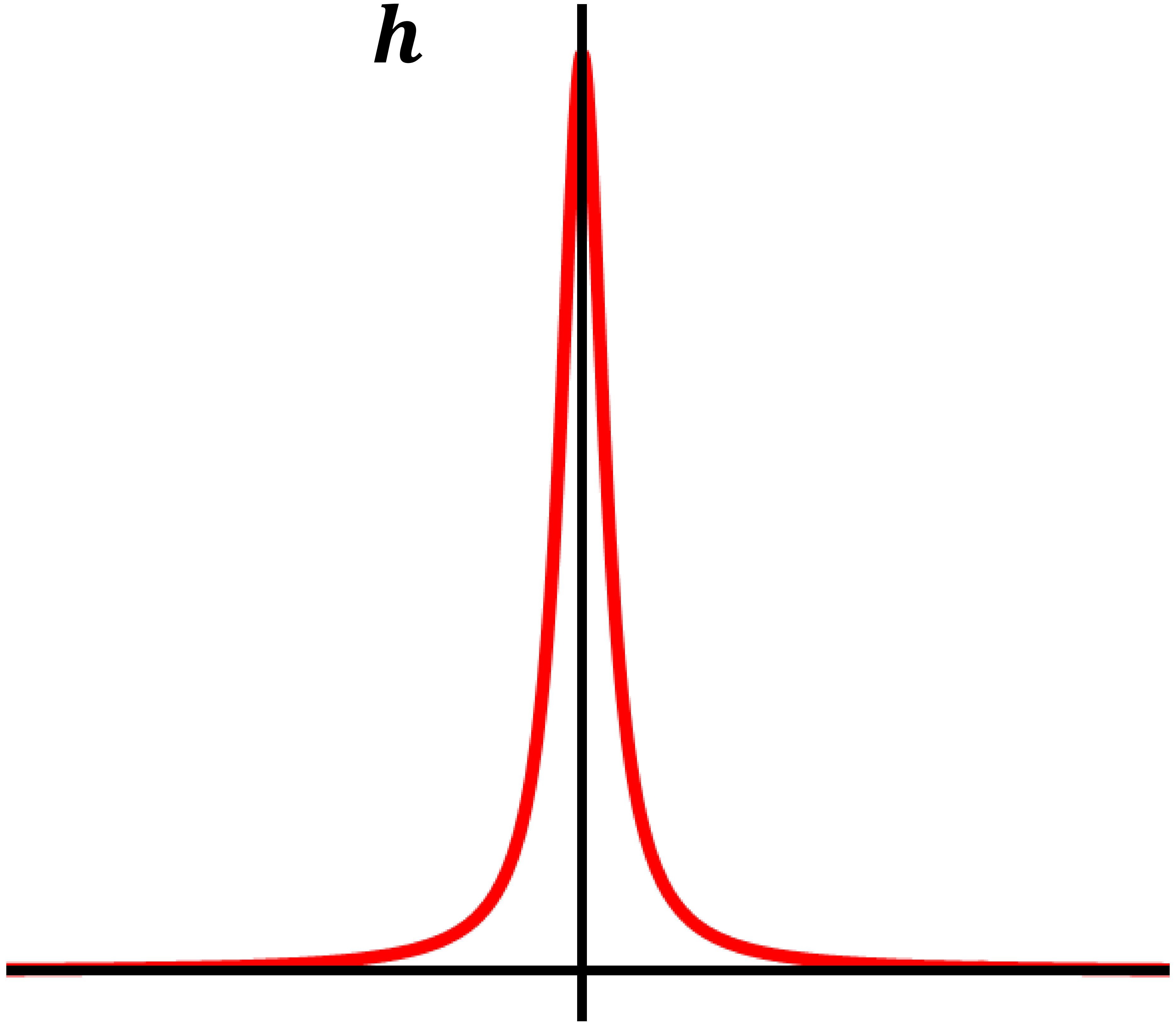
Average tangential precession



[Image source: Kadomtsev and Pogutse 1971]

- Note: we consider only most important harmonic of trapped particles; see paper in preparation for other cases

h reveals mode-particle resonance condition



- Recall: $h \sim \frac{\nu\tau_b}{(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha)^2\tau_b^2 + \nu^2\tau_b^2}$
- For most particles, $\nu^2 \ll \left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha\right)^2$
- In the limit of small ν,

$$h = -\pi \oint S[\vartheta(\tau)]P(\tau)d\tau\, \delta\left[\left(\omega - n\overrightarrow{v_d} \cdot \nabla\alpha\right)\tau_b\right]$$

- Only resonant particles participate in transport

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Resonance exists between precession and mode frequency

- h contains resonance $\delta\left[\left(\omega - n\overline{\overrightarrow{v_d}\cdot\nabla\alpha}\right)\tau_b\right]$
- Mode structure is $\overrightarrow{A_1} \sim e^{i(n\,\alpha - \omega t + (nq-m)\vartheta)}$
- Resonance occurs when particle's precession in α matches mode speed

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Integration yields diffusion coefficient

- Heat flux is given by flux surface average energy, *h,* and velocity outwards:

$$\Gamma = \left\langle \int d^3v \left(\frac{Mv^2}{2}\right) h \left(v_\parallel \hat{b} + \vec{v}_d\right)_1 \cdot \nabla\psi \right\rangle$$

- Integration gives (drop order unity numerical coefficients):

$$\Gamma \sim \sqrt{\epsilon} M c^2 \Phi^2 R^2 \Omega_p n \left(\ln \frac{n v_0^2}{\Omega_p \omega R^2}\right) \frac{\partial n_\alpha}{\partial \psi}$$

- Compare to phenomenological estimate: $\Gamma \sim -\frac{v_\psi^2}{\omega} \frac{\partial n_\alpha}{\partial \psi}$

- Logarithm comes because $\overline{\overrightarrow{v_d} \cdot \nabla\alpha}$ vanishes for some trapped particles

D_{TAE} gives condition for onset of depletion

- The diffusion coefficient resulting from flux is given by

$$\mathrm{D}_{\mathrm{TAE}} \sim n\sqrt{\epsilon}\left(\frac{Ze\Phi}{Mv_0^2}\right)^2 \rho_{p0}^2\Omega_p\left(\ln\frac{nv_0^2}{\Omega_p\omega R^2}\right)$$

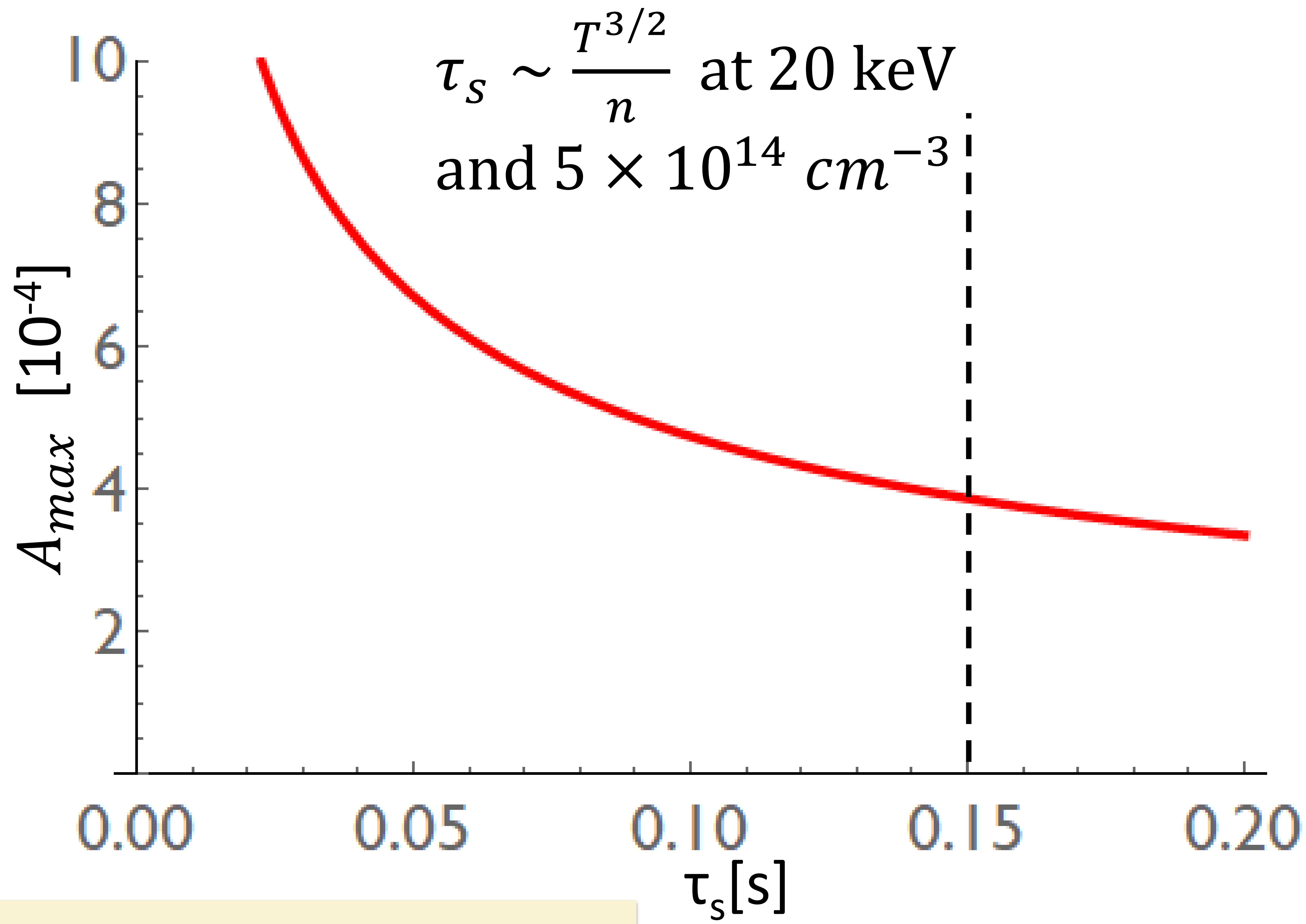
- Increases with ρ_{p0}^2, n
- Recall goal of work: determine where $\frac{D_{TAE}\tau_s}{a_\alpha^2} \sim 1$
- Significant change in alpha distribution and heating happens when

$$\frac{Ze\Phi}{Mv_0^2} \sim \left(\frac{\mathrm{a}_\alpha}{\rho_{p0}}\right)\sqrt{\frac{1}{\tau_s\Omega_p n\sqrt{\epsilon}\ln\left(\frac{nv_0^2}{\Omega_p\omega R^2}\right)}} \equiv A_{max}$$

PSFC

Only small amplitude necessary for significant redistribution

- Plot shows $\frac{D_{TAE}\tau_s}{a_\alpha^2} \sim 1$ at SPARC-like parameters
- Many studies of TAE saturation exist; some[1] show amplitudes around 10^{-4}
- TAEs could cause significant collisional redistribution
- Decreasing slowing down time (by increasing *n* and decreasing *T*) reduces effect of transport



$$A_{\max} \equiv \left(\frac{a_\alpha}{\rho_{p0}}\right)\sqrt{\frac{1}{\tau_s \Omega_p n \sqrt{\epsilon}\ln\left(\frac{n v_0^2}{\Omega_p \omega R^2}\right)}}$$

[1] See, e.g., Fitzgerald et al. NF 2016

PSFC

Conclusions

- Alpha particle physics, especially at high field, is exciting now

- Higher temperature promotes TAE instability
    - To reduce TAE instability, decrease slowing down time

  See E.A. Tolman, N.F. Loureiro, P. Rodrigues, J.W. Hughes, E.S. Marmar, *Dependence of alpha-particle-driven Alfvén eigenmode linear stability on device magnetic field strength and consequences for next-generation tokamaks*, Nuclear Fusion 59, 046020 (2019).

- Drift kinetic description of alpha transport can determine when TAEs cause significant redistribution
    - Significant transport possible
    - Transport from trapped (this presentation) and passing (paper in preparation)
    - To reduce effect of transport, decrease slowing down time

  See E.A. Tolman, P.J. Catto, N.F. Loureiro, *In preparation* (2019).