Global gyrokinetic PIC simulations for stellarators and heliotrons with emphasis on experimentally relevant scenarios

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Introduction

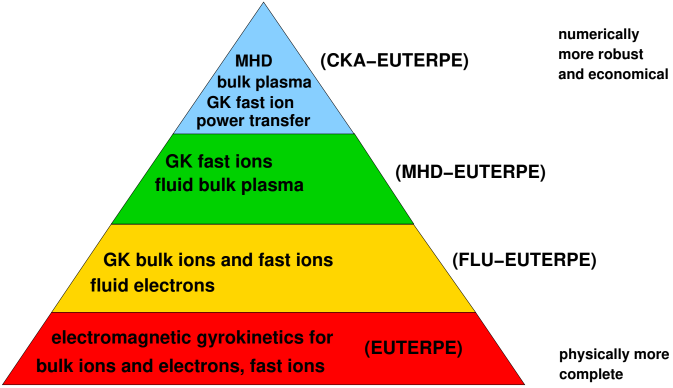
- Wendelstein 7-X has now concluded several operational phases
- perform gyrokinetic simulations for experimentally relevant scenarios
- main tool: EUTERPE
- here: main emphasis on Alfvén modes and fast particles
- nonlinear ITG simulations: see the poster by E.Sánchez on Tuesday.

Global gyrokinetic code EUTERPE

- δf particle-in-cell code
- global simulation domain: full-volume for 3D stellarator equilibria
- multiple kinetic species (ions, electrons, fast ions/impurities)
- linear/nonlinear
- electrostatic/electromagnetic (includes $\delta B_{\|}$)
- cancellation problem solved: adjustable control variate or pullback mitigation scheme
- arbitrary wavelength (Padé approximation)
- linearised collision operators with moment conservation (e.g. pitch angle, slowing down)
- multiple distribution functions (Maxwellian, slowing-down, ...)

Hierarchy of models

Fully gyrokinetic simulations are very time-consuming and difficult.
Simplified models sacrifice physics details for gain in speed.
$\Rightarrow$ EUTERPE, FLU-EUTERPE, MHD-EUTERPE, CKA-EUTERPE



CKA: Code for Kinetic Alfvén waves

Perturbative MHD hybrid model (CKA-EUTERPE)

- assume linear eigenmode $\{\phi_0(\vec{r}), \omega\}$ (e.g. from MHD code CKA) and allow for slowly varying amplitude $\hat{\phi}(t)$ (similar for $A_\parallel$):

$$\phi = \mathrm{Re}[\hat{\phi}(t)\,\phi_0(\vec{r})\,\exp(\mathrm{i}\omega t)]$$

- amplitude equations (with external damping γ_d)

$$\frac{\partial\hat{\phi}\,(t)}{\partial t} = \mathrm{i}\omega\left(\hat{A}_\parallel - \hat{\phi}\right) + 2\left(\gamma\,(t) - \gamma_\mathrm{d}\right)\hat{\phi}$$

$$\frac{\partial\hat{A}_\parallel\,(t)}{\partial t} = \mathrm{i}\omega\left(\hat{\phi} - \hat{A}_\parallel\right)$$

- mode growthrate $\gamma = P(t)/T$ from wave-particle energy transfer $P(t)$

$$P(t) = -\int \mathrm{d}W\mathrm{d}V\, f_\mathrm{fast}^{(1)}\left[\frac{1}{B}\vec{b}\times(mv_\parallel^2\vec{\kappa} + \mu\nabla B)\cdot\nabla\phi^*\right]$$

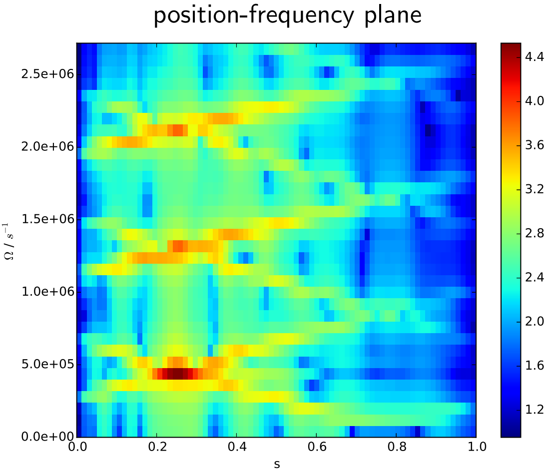
$$T = \int \mathrm{d}V\,\frac{Mn}{B^2}|\nabla_\perp\phi|^2$$

- use $\phi, A_\parallel$ as fields in the gyrokinetic equation for fast particles

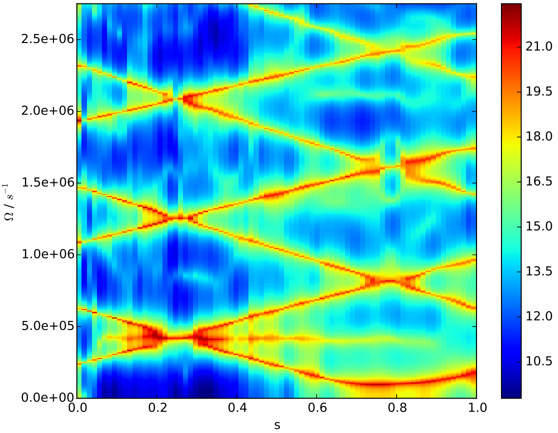
Allows very fast simulation of nonlinear behaviour (our workhorse for investigating fast particle interaction).

FFT or periodogram

- standard methods
- very long time traces necessary for good frequency resolution
- for statistical signals the variance does not reduce
- example: get Alfvén continuum from gyrokinetic simulation of tokamak
  $\Rightarrow$ FFT for each radial channel $(s = (r/a)^2)$



position-frequency plane



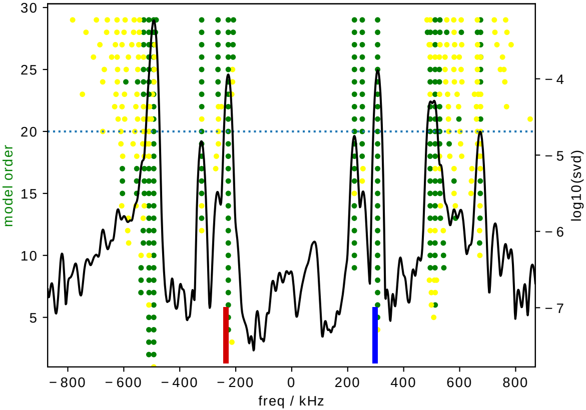
Parametric methods:

- Presuppose a given signal model and estimate its parameters (super-resolution methods)
- Damped MUltiple SIgnal Classification (DMUSIC)

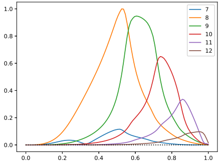
⇒ highly improved frequency resolution for the same dataset

Common problem in simulations: presence of many modes

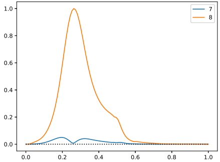
SSI: Draw conclusions about a hidden linear system by some of its observables corrupted by noise (similar to hidden Markov models). Mode frequencies/structures from time signals at different radii.



toroidal Alfvén eigenmode



reversed shear Alfvén eigenmode

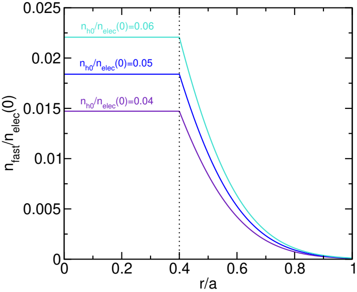


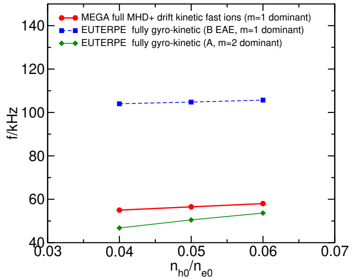
1. EUTERPE
2. Signal processing tools for diagnostics of simulation datasets
3. **Comparison of EUTERPE (EU) with MEGA (Japan)**
4. Alfvénic dynamics and fast particles in W7-X
5. Current Crashes in W7-X
6. Drift instabilities in W7-X for realistic profiles

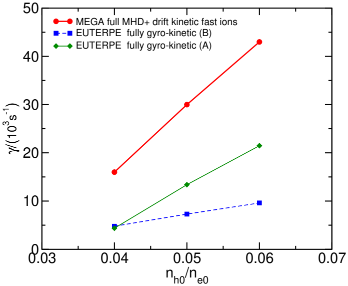
Comparison of EUTERPE and MEGA

- compare codes for fast particle driven Alfvén modes
  - MEGA: resistive full nonlinear MHD, kinetic fast ions
  - EUTERPE: full GK
- Large Helical Device (LHD) case: $B_0 = 0.619\text{T}, R_0 \approx 3.7\text{m}, \langle\beta\rangle \approx 3\%$ $T_\text{i} = T_\text{e} = 1\text{keV}, T_\text{fast} = 100\,\text{keV}$
- sequence of specially tailored fast particles density profiles: $\beta_\text{fast}(0) \approx 1.3\%,\ 1.625\%,\ 1.95\%$
- look for modes with $m = 0 \ldots 4, n = -1$

fast particle density







- EUTERPE gets mixture of modes, necessary to separate with e.g. SSI $\Rightarrow$ dominant energetic particle mode (EPM) also found by MEGA, subdominant elliptical Alfvén eigenmode (EAE)
- fully GK calculation leads to kinetic bulk damping not present in MHD
- good agreement with MEGA (apart from kinetic bulk damping which may be emulated by resistivity)

Geometry issues for EUTERPE solved in principle: GK treatment of centre and low m modes seems to be fine.
Next step: a detailed benchmark with MEGA is in preparation.

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Periodic chirping in W7-X using a Krook operator

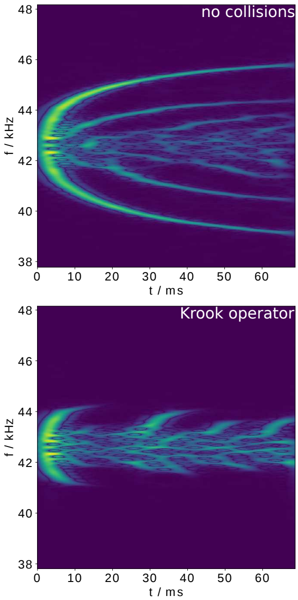
- frequency chirping investigated in W7-X by nonlinear CKA-EUTERPE simulation of toroidal Alfvén mode
- chirping as an inherent nonlinear effect seen in the time-frequency domain
- experiments often show periodically recurring chirping
- in contrast to Berk-Breizman paradigm pitch-angle collisions were found not to be sufficient to rebuilt the distribution function

$\Rightarrow$ velocity-space versus real-space gradients

- a required particle source, rebuilding the distribution function, is emulated here using a Krook operator

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \mathcal{C}_{\mathrm{K}} = -\nu_{\mathrm{K}}\left(f - f^{(0)}\right)$$

- several 10 ms of mode development can be simulated

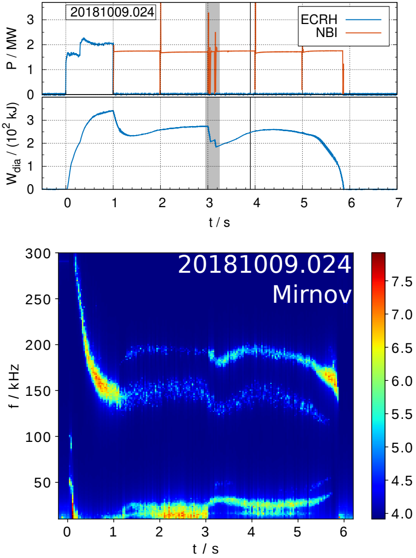


Alfvénic dynamics in W7-X (shot 20181009.024)

- last operation phase of W7-X (July to October 2018) for the first time featured NBI

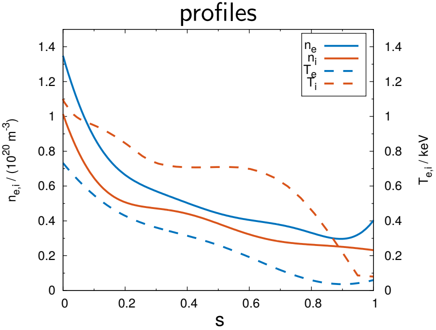
⇒ fast ions (55 keV) were present in the machine

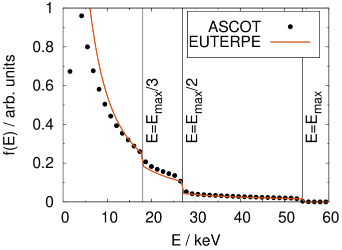
- can fast ions resonantly excite Alfvén eigenmodes in W7-X ?
- NBI-dominated discharge with ECRH start-up, NBI source (1.75 MW) switched each second
- mode activity observed in some shots with Mirnov, phase contrast imaging (PCI) and X-ray tomography (XMCTS)

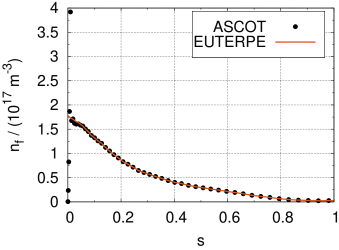


20181009.024: profiles

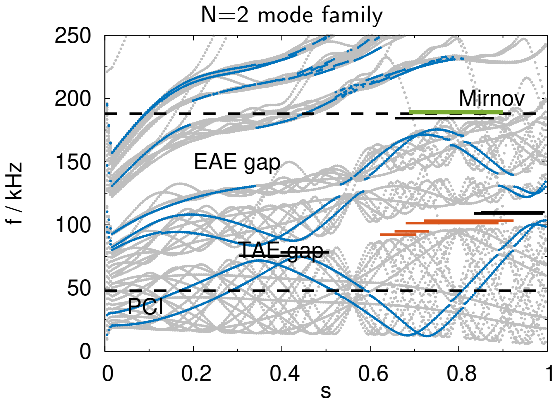
- characteristic for NBI discharges in W7-X: strong fuelling $\Rightarrow$ density peaks on axis
- radial electric field taken into account (minor correction)
- anisotropic ASCOT distribution function (S.Äkäslompolo) used to fit an isotropic model distribution function







20181009.024: Alfvén continuum and modes



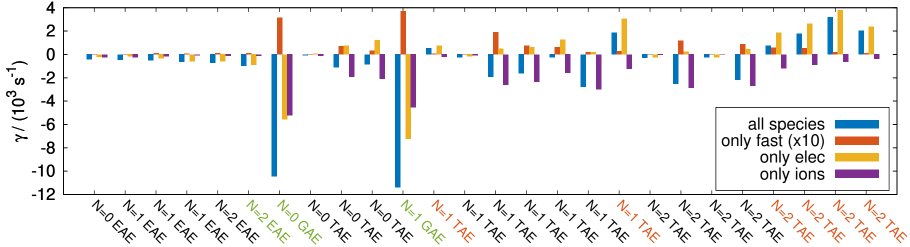
- CKA found 26 global Alfvén eigenmodes: global (GAE), toroidal (TAE), elliptical (EAE)
- SSI analysis of Mirnov data suggests modes with m ∈ [10, 15]

⇒ frequency of core-localized GAEs ($N = 0$ family) matches those from PCI and XMCTS

- TAEs at around 100 kHz were not observed by any diagnostics

Can these modes be destabilized by fast ions?

CKA-EUTERPE gives an inconclusive picture on growthrates:



- kinetic effects of background-plasma dominate those by fast particles (understandable since fast-ion β is low)
- some TAEs are destabilized by background-plasma electrons (but not observed by the diagnostics, react sensitively on profiles)

⇒ core-localized GAE react most strongly to fast ions, but also have the strongest damping rates, their frequencies agree with experimental measurements

- sensitivity check on profiles gives similar conclusions for GAE

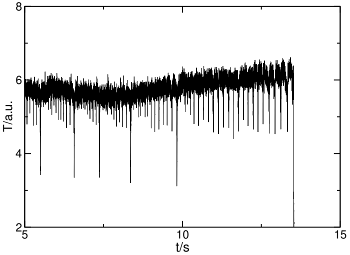
Simulations in LHD are possible. Try application to the geometrically much more complex W7-X.

- ECCD experiments: sawteeth-like current crash events related to $\iota \approx 1$ observed: cause is unclear
- different crash events seen in experiment: centre $\Leftrightarrow$ edge
- parallel current by itself can drive instabilities
- $I_{||} \sim 10$ MA in tokamaks vs. $I_{||} \sim 10$ kA in optimised stellarators

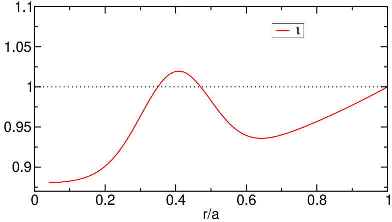
$\Rightarrow$ confinement not lost in stellarators even if parallel plasma current disappears

- determine pre-crash linear scenario: which instability?

ECE signal (M.Zanini)



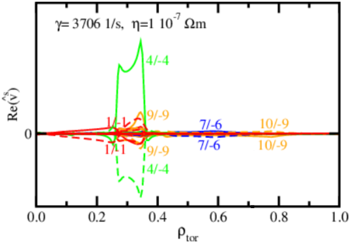
typical ECCD rotational transform

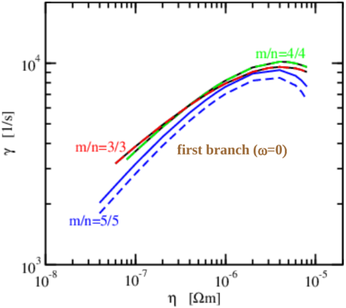


Current crashes - 3D resistive MHD

First attempt: look for MHD instability

- no ideal MHD instability found
- tearing mode found with resistive MHD (CASTOR3D)

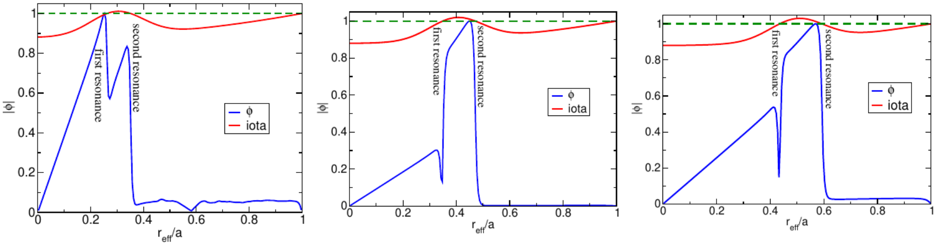




- resistivity of W7-X ($\eta \approx 5 \cdot 10^{-9}\,\Omega$m) much lower than in the modelling

$\Rightarrow$ this mode is possibly not responsible for the current crashes

Gyrokinetic simulation for a cylinder



- double kink mode found
- shifting $\iota = 1$ position outwards increases growth rate
- a hint on what is going on in W7-X: kinetic effects ?

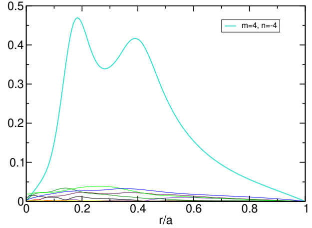
Gyrokinetic simulation for W7-X

- typical profile shape used,
  $n_e = 2 \cdot 10^{19}\,\mathrm{m}^{-3}$,
  $T_{\mathrm{e},0} = 5\,\mathrm{keV}, T_{\mathrm{i},0} = 2\,\mathrm{keV}$
- fully gyrokinetic calculation with EUTERPE
- Two dominant modes were found: kink mode and pressure driven mode (also seen in tokamak)

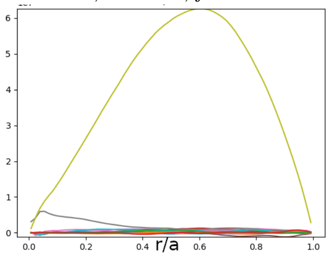
$\Rightarrow$ Inconclusive because of sensitivity on equilibrium:
Unclear numerical problems related to VMEC equilibria near axis may drive instability.

- See also the poster by A.Zocco on Wednesday.

$m = -4, n = 4, f \approx 10\mathrm{kHz}$



$m = 1, n = -1, f \approx 40\mathrm{kHz}$



Wendelstein 7-X

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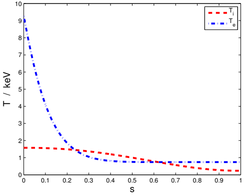
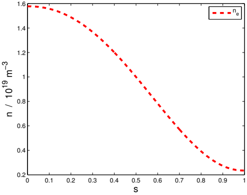
GK simulation for a W7-X discharge

operational phase 1.1 scenario

temperatures



electron density

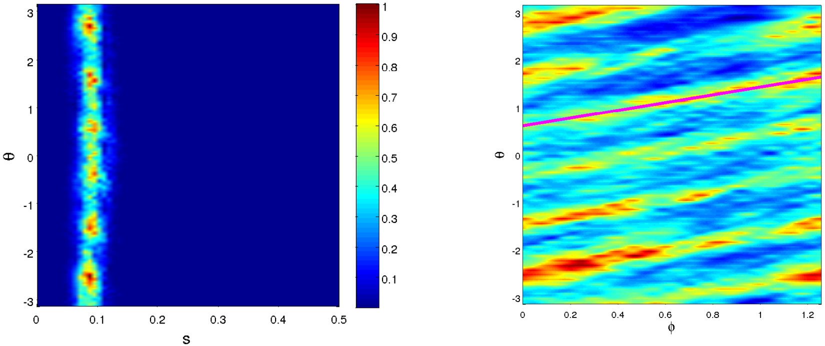


- discharge where hot electrons are the dominant species
- strong radial electric field with strong shear

Electrostatic simulation with kinetic electrons ($k_\perp \rho_\mathrm{i} \approx 2.4$, f $\approx 450\,\mathrm{kHz}$)

modulus of electrostatic potential

$\varphi = 0.0$ s=0.1



- clearly driven by electrons
- phase space diagnostics $\Rightarrow$ not a TEM ($\omega > 0$)
- try to clarify reason for distinctive mode structure

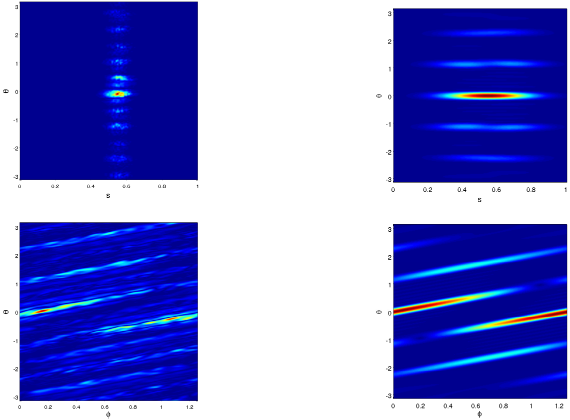
ITG mode structure in W7-X

- construct a semi-empirical model to generate mode structure
- model assumes ι-profile and a few free parameters
  $\Rightarrow$ no information about equilibrium coupling necessary

simulation result — semi-empirical model

s-θ plane

ϕ-θ plane



- work in progress: application to electron driven mode

Conclusions

- EUTERPE: Simulation of electromagnetic modes driven by fast particles now possible due to recent numerical developments.
- MHD hybrid model allows fast and robust simulation of fast particle driven modes ⇒ possibly no NBI driven modes in W7-X.
- Full gyrokinetic electromagnetic simulations for LHD are now possible. For W7-X numerical problems remain.
- Electron driven electrostatic drift mode for experimental scenario found.
- Simulation tools are mature for application to experimental results but better data from experiment are needed.

Additional Material

Full gyrokinetic model

Derived from a Lagrangian:

$$L = \sum_{\text{species}} \int \left\{ f \left[(q\vec{A} + p_\parallel \vec{b}) \cdot \dot{\vec{R}} + \frac{m^2}{q} \mu \dot{\alpha} - \frac{1}{2m} p_\parallel^2 - m\mu B - q\langle \phi \rangle + \frac{q}{m} p_\parallel \langle A_\parallel \rangle \right] + f_0 \left[\frac{m}{2B^2} (\nabla_\perp \phi)^2 - \frac{q^2}{2m} \langle A_\parallel \rangle^2 \right] \right\} \mathrm{d}V \mathrm{d}W - \frac{1}{2\mu_0} \int (\nabla_\perp A_\parallel)^2 \mathrm{d}V$$

Electromagnetic simulations in $p_\parallel$ are hampered by the cancellation problem:

- first moment of $p_\parallel$ does not give a physical current
- Ampère's law obtains an unphysical skin term which cancels with the adiabatic part from the current

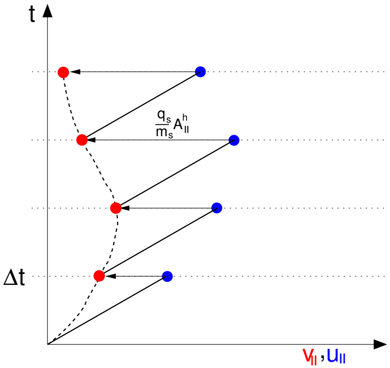
$\Rightarrow$ numerical cancellation of two large terms represented differently (matrix $\Leftrightarrow$ particles)

Cancellation problem mitigated by adjustable control variate method (ACV). (Hatzky et al. 2007)

Full GK: electromagnetic simulations

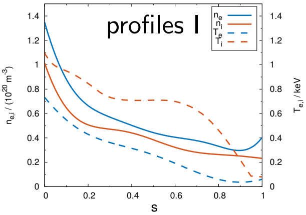
ACV superseded by pullback scheme introduced by Mishchenko et al. 2014:

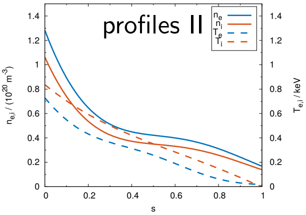
- start from $v_\parallel$
- use splitting $A_\parallel = A_\parallel^{\mathrm{s}} + A_\parallel^{\mathrm{h}}$ and introduce $u_\parallel = v_\parallel + \frac{q}{m} A_\parallel^{\mathrm{h}}$
- simplify equations by using the resulting freedom to postulate a new field equation $\frac{\partial A_\parallel^{\mathrm{s}}}{\partial t} + \nabla_\parallel \phi = 0$
- combine with restarting scheme
  - after some time steps: $A_\parallel^{\mathrm{s}} + A_\parallel^{\mathrm{h}} \rightarrow A_\parallel^{\mathrm{s}}$ and set $A_\parallel^{\mathrm{h}} = 0$
  - transform f from $u_\parallel$-space to $v_\parallel$-space



- effectively an $v_\parallel$ simulation
- scheme allows for much larger time steps than control variate method
- enables electromagnetic simulations in parameter regimes which were not accessible before

Sensitivity on experimental profiles fit





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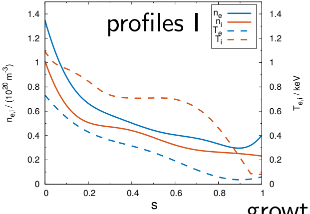
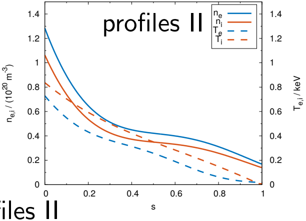
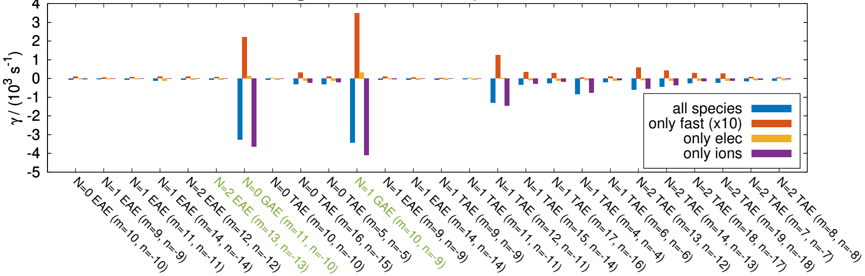




growthrates for profiles II



- again GAE most strongly destabilized by fast particles
- no clear evidence for fast particle driven modes
- CKA-EUTERPE: robust and fast, but reliable experimental data needed