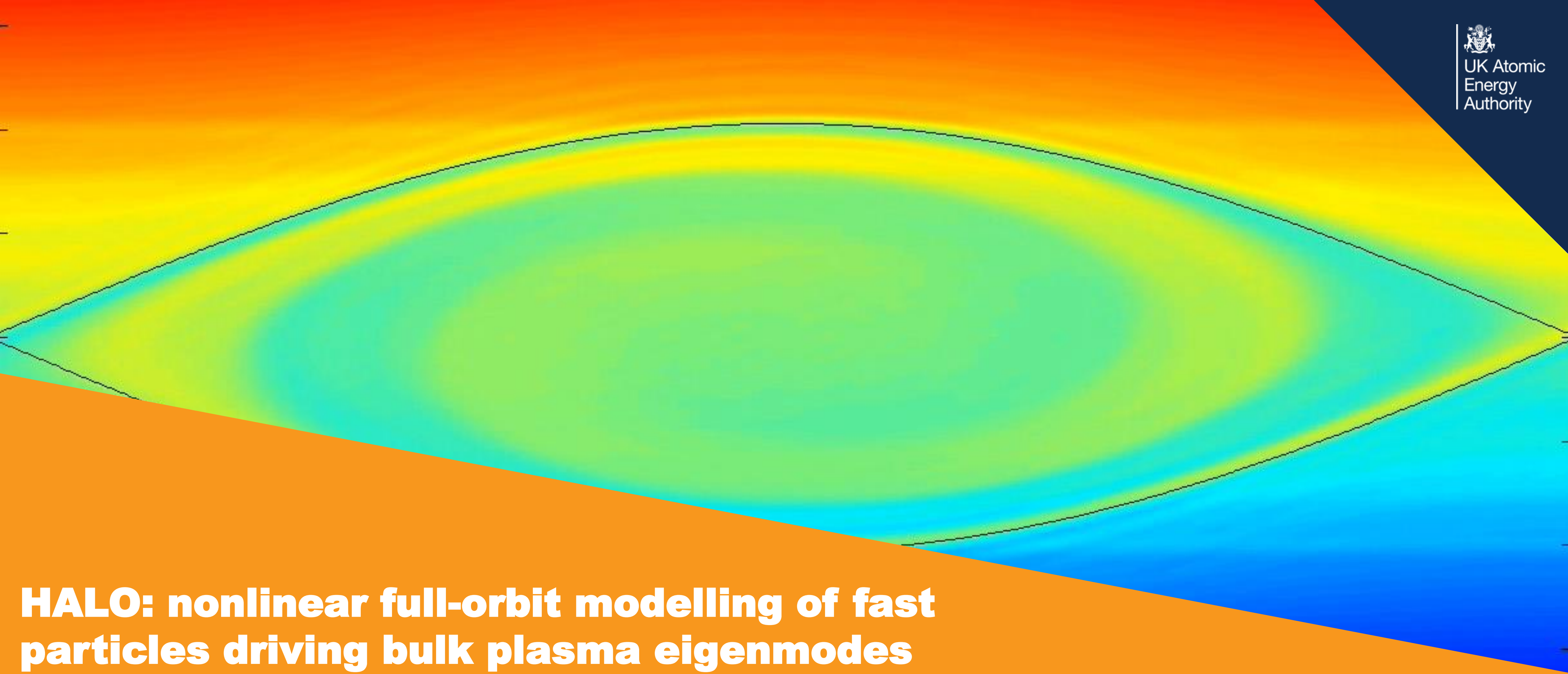

UK Atomic Energy Authority

HALO: nonlinear full-orbit modelling of fast particles driving bulk plasma eigenmodes

M. Fitzgerald, J. Buchanan, R. Akers, B.N. Breizman, S. Sharapov

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[Grant number EP/P012450/1]

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Overview

- Review of the classic electrostatic wave-particle problem for a single wave
- Derivation of the equations in HALO
  - Maxwell equations as applied to the TAE problem
  - Vlasov delta-f equations
- Representing the waves
- Representing the fast ions
- Demonstration of classic behaviour
- Generalization to other wave-particle problems

On Landau Damping

John Dawson

Let us return now to the problem of Landau damping and assume that the electric field has the form (10)

$$E_1 = E_1 \sin(kx - \omega t). \tag{10}$$

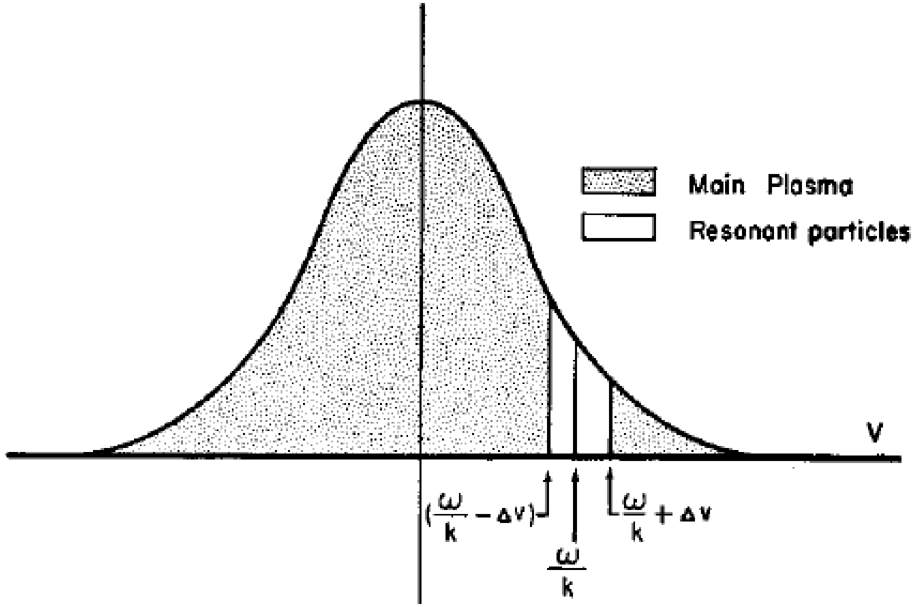
This field is to be produced by a normal mode of the main plasma. On substituting (10) in the equations of motion, (1) and (2), we find that n_1 and v_1 for the main plasma are given by (11) and (12).

$$v_1(x, t) = -\frac{eE_1}{m(\omega - kV)} \cos(kx - \omega t) \tag{11}$$

$$n_1(x, t) = -\frac{eE_1 N(V) k}{m(\omega - kV)^2} \cos(kx - \omega t). \tag{12}$$

Further, we assume that the electric field is produced by the main plasma and that the resonant particles make a negligible contribution to it. This is equivalent to assuming small damping. On substitution

Dawson, J. (1961). On Landau Damping. *Physics of Fluids*, *4*(7), 869. doi:10.1063/1.1706419



$$1 = +\frac{4\pi e^2}{m} \int_{mp} \frac{N(V)\, dV}{(\omega - kV)^2}.$$

Here mp means the integral is to be taken only over the main plasma. Since the resonant particles have been cut out no problem with singular integrals arises here.

$$\partial(E_1^2/8\pi)/\partial t = -E_1 \cdot j_1.$$

$$\frac{dE_1}{dt} = \frac{\pi\omega}{2}\left(1 - \frac{k}{\omega}\frac{d\omega}{dk}\right)\frac{\omega_p^2}{k^2} f'\left(\frac{\omega}{k}\right) E_1. \tag{31}$$

Equation (31) is the damping one obtains by Landau's formalism.

- For sufficiently weak resonant wave-particle interaction, a minority of particles are responsible for the majority of the drive
- This is the assumption of the classic linear Landau damping
- The drive can be expressed as a power transfer

Nonlinear behaviour of resonant minority

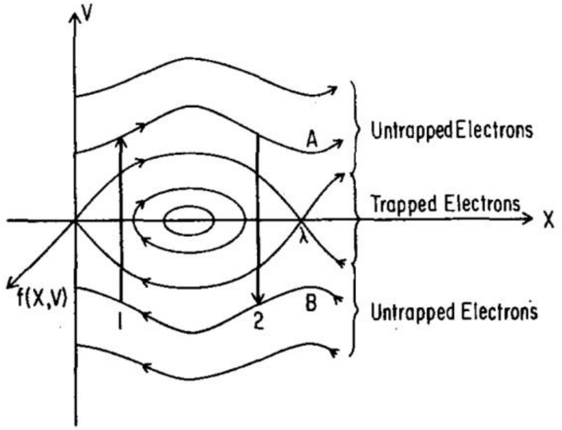
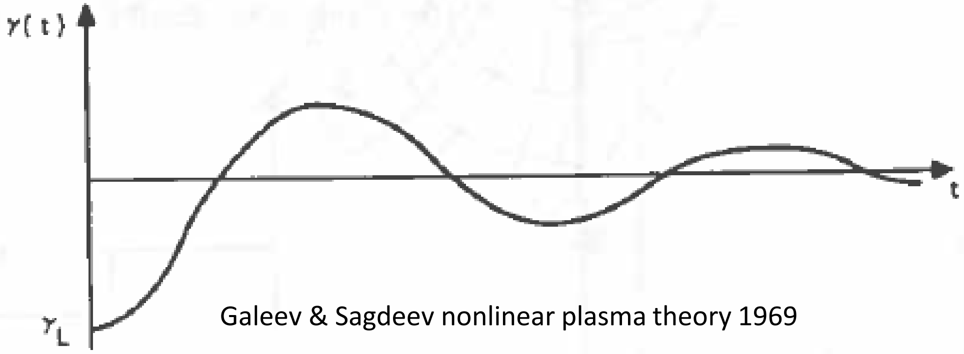


FIG. 2. The phase trajectories of the resonant electrons.

O'Neil T 1965 Collisionless Damping of Nonlinear Plasma Oscillations Phys. Fluids 8 2255

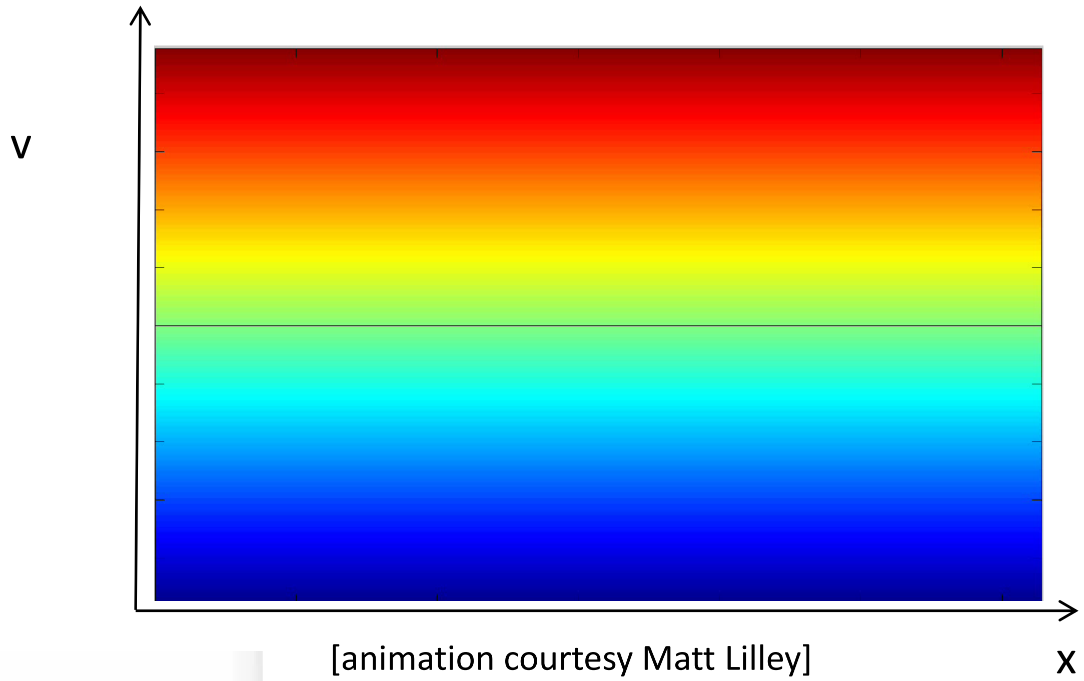
It is convenient to introduce a time-dependent damping coefficient

$$\gamma(t) \equiv \frac{1}{2\mathcal{E}(t)} \frac{d}{dt} \mathcal{E}(t),$$

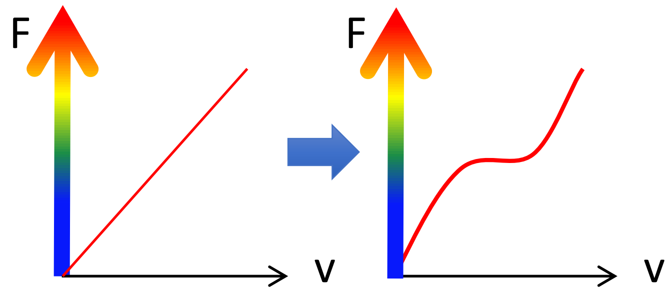


Galeev & Sagdeev nonlinear plasma theory 1969

FIGURE II-4. Time-dependent damping coefficient.



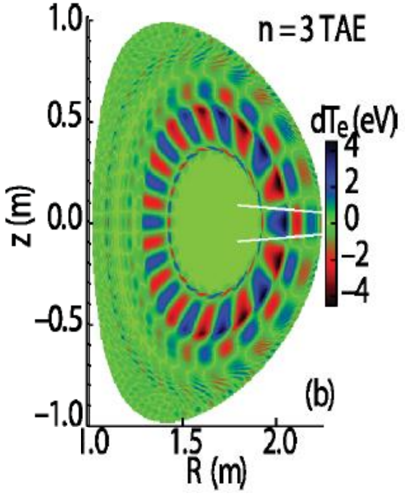
[animation courtesy Matt Lilley]

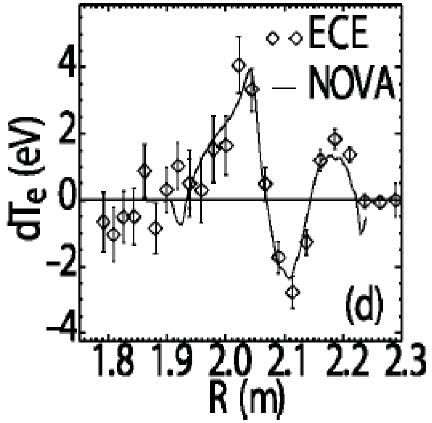


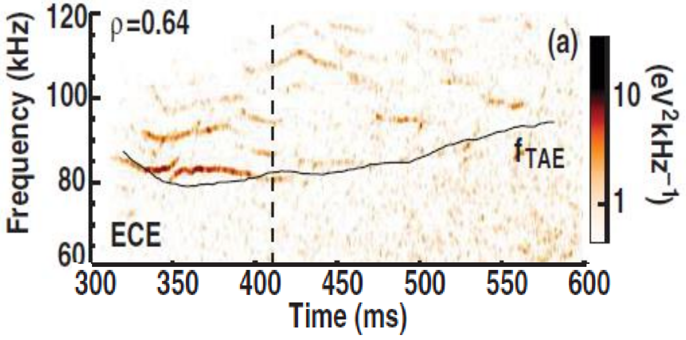
TAEs in experiment closely resemble MHD linear eigenmodes

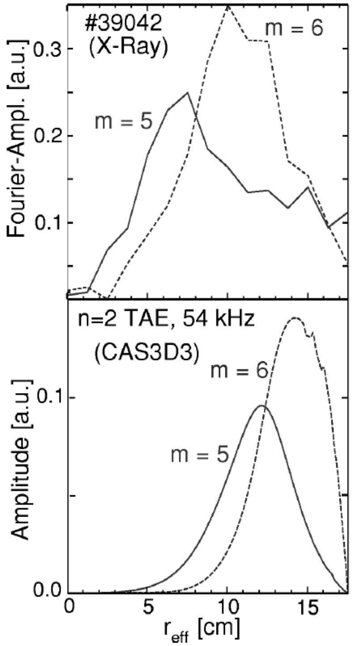
- The toroidal Alfven eigenmode TAE is a discrete normal mode solution of the MHD linear system of equations in a tokamak
- The drive of TAEs observed in experiment is due to resonant fast particles present due to NBI, ICRH and alpha heating
- Eigenmode structure and frequency very close to MHD solution even though fast particles are responsible for the drive

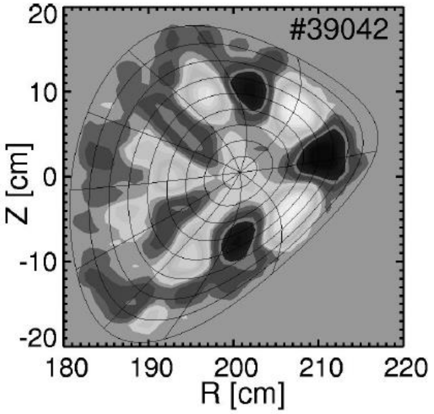
DIII-D tokamak
[VanZeeland 2006]











W7-AS stellarator
[Weller 2001]

Van Zeeland, M. A. et al. (2006). *Physical Review Letters*, *97*(13), 135001. doi:10.1103/PhysRevLett.97.135001
Weller, A. et al. (2001). *Physics of Plasmas*, *8*(3), 931. doi:10.1063/1.1346633

Identifying currents in Maxwell's wave equation

$$-\frac{c^2}{\omega^2}\nabla\times\nabla\times\tilde{\mathbf{E}}(\mathbf{x},\omega)+\tilde{\mathbf{E}}(\mathbf{x},\omega)=-\frac{i\mu_0 c^2}{\omega}\tilde{\mathbf{J}}(\mathbf{x},\omega)$$

$$-\frac{c^2}{\omega^2}\nabla\times\nabla\times\tilde{\mathbf{E}}(\mathbf{x},\omega)+\tilde{\mathbf{E}}(\mathbf{x},\omega)$$
$$=-\frac{i\mu_0 c^2}{\omega}\left[\int d\boldsymbol{x}'\,\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\boldsymbol{x}',\omega)\;+\tilde{\mathbf{J}}_{\boldsymbol{NL}}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{\overline{\sigma}}(\mathbf{x},\omega)\right]$$

$\int d\boldsymbol{x}'\,\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\boldsymbol{x}',\omega)$ is the sloshing of tiny waves which don't decay in time (the linear Hermitian response)
$\tilde{\mathbf{J}}_{\overline{\sigma}}(\mathbf{x},\omega)$ is the decay of tiny waves (the linear anti-Hermitian response)
$\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)$ are any imposed currents that don't depend on the electric field (antennas, act of God)
$\tilde{\mathbf{J}}_{\boldsymbol{NL}}(\mathbf{x},\omega)$ are any nonlinear response currents

Why TAEs resemble linear MHD solutions

$$-\frac{c^2}{\omega^2}\nabla\times\nabla\times\tilde{\mathbf{E}}(\mathbf{x},\omega)+\tilde{\mathbf{E}}(\mathbf{x},\omega)$$
$$=-\frac{i\mu_0 c^2}{\omega}\left[\int dx'\,\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\boldsymbol{x}',\omega)\;+\tilde{\mathbf{J}}_{NL}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{\bar{\sigma}}(\mathbf{x},\omega)\right]$$

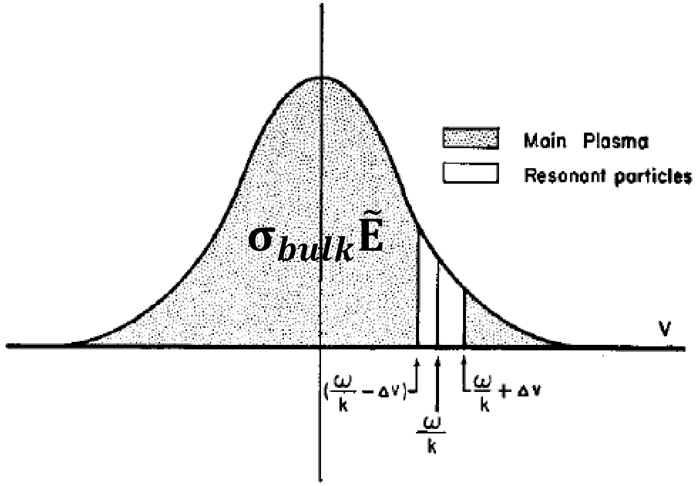
The shear Alfven wave response in cold plasma and $\omega \ll \Omega_c$ is due to the polarization drift of ions

$$\boldsymbol{v}_{\boldsymbol{bulk}}(\boldsymbol{x},t)=\frac{m_i}{e_i(\boldsymbol{B}+\delta\boldsymbol{B})^2}\frac{d\boldsymbol{E}}{dt}$$

This bulk fluid motion has an associated nonlinearity

$$\frac{\left|\tilde{\mathbf{J}}_{\boldsymbol{NL,bulk}}\right|}{\left|\boldsymbol{\sigma}_{\boldsymbol{bulk}}\tilde{\mathbf{E}}\right|}=\frac{\delta\boldsymbol{B}^2}{\boldsymbol{B}^2}+\cdots$$

The “fluid nonlinearity” from plasma sloshing is thus tiny $\frac{\delta\boldsymbol{B}}{\boldsymbol{B}}\sim 0.1\%$

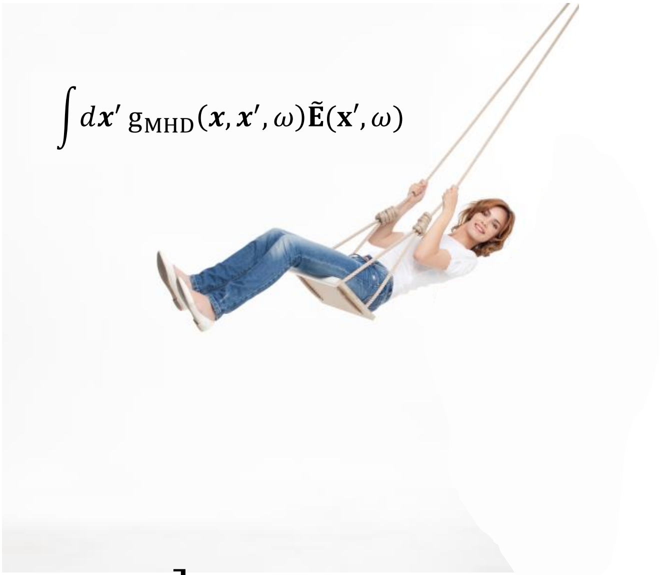


The main nonlinearity is then the behaviour of the resonant population

The undamped oscillation of a plasma

Suppose you have a plasma that you perturb a little bit from equilibrium that undergoes undamped oscillations

$$-\frac{c^2}{\omega^2}\nabla\times\nabla\times\tilde{\mathbf{E}}(\mathbf{x},\omega)+\tilde{\mathbf{E}}(\mathbf{x},\omega)$$
$$=-\frac{i\mu_0 c^2}{\omega}\left[\int d\boldsymbol{x}'\,\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\boldsymbol{x}',\omega)+\tilde{\mathbf{J}}_{NL}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{\bar{\sigma}}(\mathbf{x},\omega)\right]$$



$\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{x}',\omega)$ can be any Hermitian (oscillating fluid) model of the plasma such as ideal MHD or the cold plasma equations

$$\left(\frac{i\mu_0 c^2}{\omega}\right)^{-1}\int d\boldsymbol{x}'\left[-\frac{c^2}{\omega^2}\boldsymbol{\delta}(\boldsymbol{x}-\boldsymbol{x}')\nabla\times\nabla\times+I+\frac{i\mu_0 c^2}{\omega}\boldsymbol{\sigma}_{\boldsymbol{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega)\right]\tilde{\mathbf{E}}(\mathbf{x}',\omega)$$
$$=0+\tilde{\mathbf{J}}_{NL}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{\bar{\sigma}}(\mathbf{x},\omega)$$

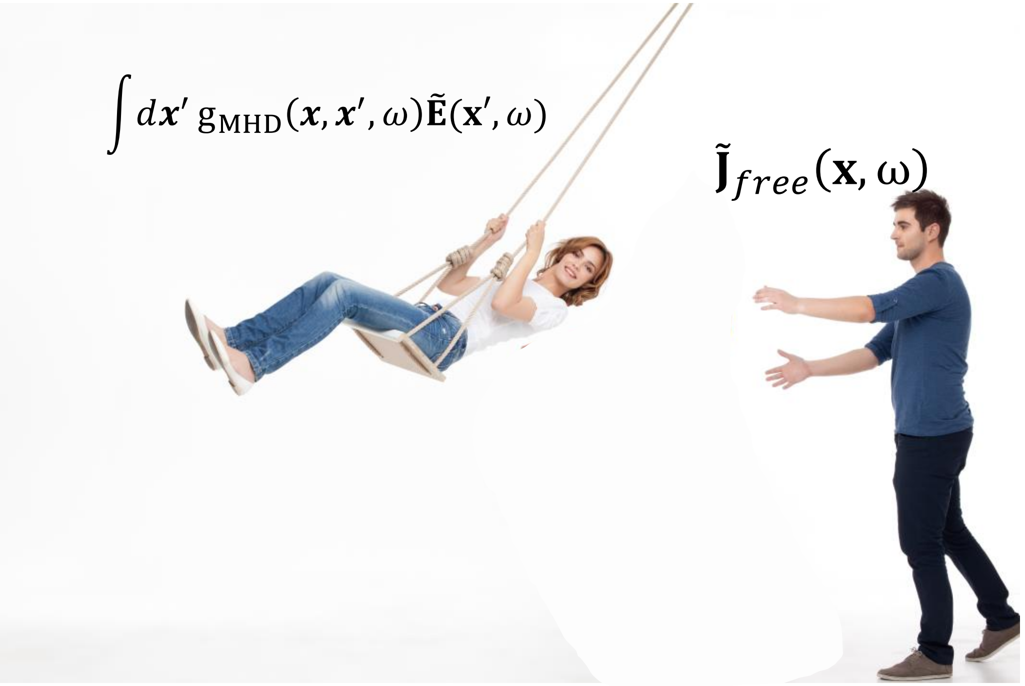
$$\int d\boldsymbol{x}'\,\mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\mathbf{x}',\omega)=0+\tilde{\mathbf{J}}_{NL}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)+\tilde{\mathbf{J}}_{\bar{\sigma}}(\mathbf{x},\omega)$$

A weak drive of the oscillation looks like the undriven solution plus a correction

$$\int d\boldsymbol{x}'\, \mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\mathbf{x}',\omega) = 0$$

If the drive is weak, then most of the currents are in the sloshing not in the drive

The electric field will look almost like an MHD solution



$$\frac{|\omega-\omega'|}{|\omega|} \sim \frac{\left|\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)\right|}{\left|\int d\boldsymbol{x}'\, \boldsymbol{\sigma}_{\boldsymbol{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\mathbf{x}',\omega)\right|} \ll 1$$

$$\int d\boldsymbol{x}' \left[\mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega') + \epsilon\left[(\omega-\omega')\frac{\partial \mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega')}{\partial\omega}\right]\right]\tilde{\mathbf{E}}(\mathbf{x}',\omega) = \epsilon\tilde{\mathbf{J}}_{free}(\mathbf{x},\omega)$$

M. Fitzgerald | 18th European Fusion Theory Conference, Ghent | 7 October 2019

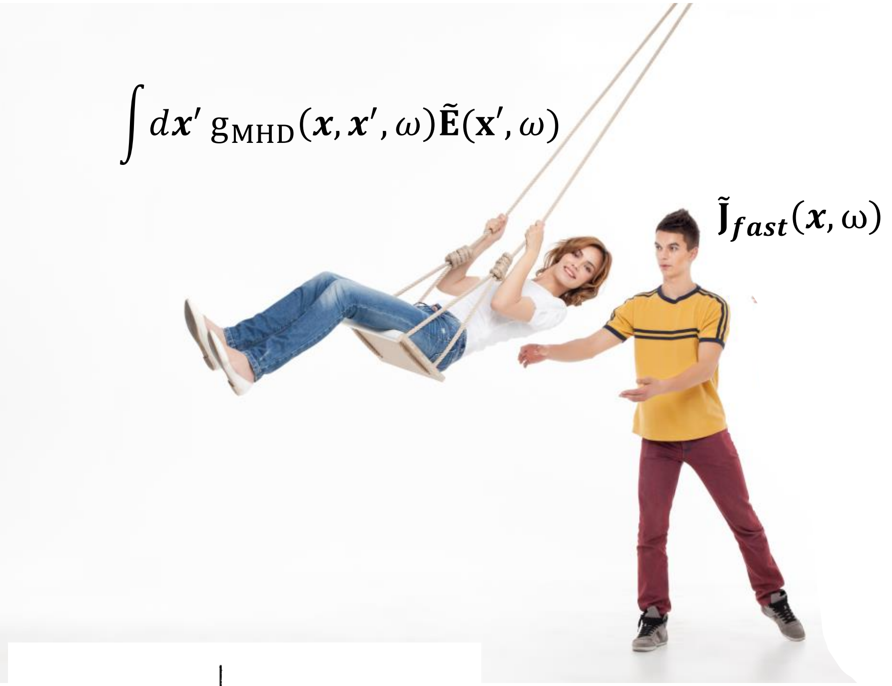
TAEs are MHD waves in the *bulk plasma* that are *driven* by resonant fast particles

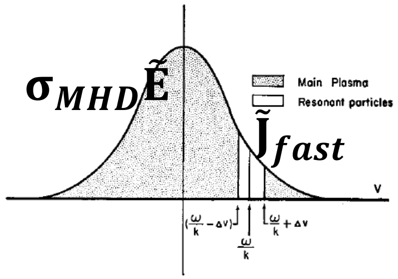
The TAE is an eigenmode of the ideal MHD problem

$$\int d\boldsymbol{x}'\, g_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega)\tilde{\mathbf{E}}(\mathbf{x}',\omega) = 0$$

TAEs are driven weakly in experiment $\frac{|\omega-\omega'|}{|\omega|} \sim \frac{\gamma}{\omega} \sim 1\%$, meaning that the resonant population is a small compared with the bulk

$$\int d\boldsymbol{x}' \left[g_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega') + \left[(\omega-\omega')\frac{\partial}{\partial\omega} \right] g_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega') \right] \tilde{\mathbf{E}}(\mathbf{x}',\omega) = \tilde{\mathbf{J}}_{fast}(\boldsymbol{x},\omega)$$





Solution of the wave equation

$$\int d\boldsymbol{x}' \left[\mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega') + \left[(\omega-\omega')\frac{\partial}{\partial\omega}\right] \mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega') \right] \tilde{\mathbf{E}}(\mathbf{x}',\omega) = \tilde{\mathbf{J}}_{\boldsymbol{fast}}(\boldsymbol{x},\omega)$$

Exploiting that $\mathrm{g}_{\mathrm{MHD}}$ is Hermitian gives an ODE for the mode amplitude

$$\dot{A}(t;\omega_j) = -\frac{1}{2\delta W_{MHD}} \boldsymbol{e}^{\boldsymbol{i}\omega_j \mathrm{t}} \int d\boldsymbol{x}\, \boldsymbol{e}^{\dagger}(\boldsymbol{x};\omega_j)\delta \boldsymbol{J}_{\boldsymbol{fast}}(\mathbf{x},\mathrm{t})$$

$$\delta W_{MHD} = -\frac{i}{4}\int d\boldsymbol{x} d\boldsymbol{x}'\, \boldsymbol{e}^{\dagger}(\boldsymbol{x};\omega_j)\frac{\partial \mathrm{g}_{\mathrm{MHD}}(\boldsymbol{x},\boldsymbol{x}',\omega_j)}{\partial\omega}\, \boldsymbol{e}(\boldsymbol{x}';\omega_j)$$

$$\delta\boldsymbol{E}(\boldsymbol{x},t;\omega_j) = Re\{A(t;\omega_j)\boldsymbol{e}(\boldsymbol{x};\omega_j)e^{-i\omega_j t}\}$$

Completely analogous to Dawson result

It is convenient to introduce a time-dependent damping coefficient

$$\gamma(t) \equiv \frac{1}{2\mathcal{E}(t)}\frac{d}{dt}\mathcal{E}(t),$$

$$\partial(E_1^2/8\pi)/\partial t = -E_1 \cdot j_1 .$$

Representing the nonlinear fast ion current

$$\delta \boldsymbol{J}_{\boldsymbol{fast}}(\boldsymbol{x},t) = \int d\boldsymbol{v}\, f(\boldsymbol{x},v,t) Ze\boldsymbol{v}$$

The nonlinear resonant fast ions may be represented by the Vlasov equation

$$\frac{\partial f(\boldsymbol{x},\boldsymbol{v},t)}{\partial t} + v \cdot \frac{\partial f}{\partial \boldsymbol{x}}(\boldsymbol{x},\boldsymbol{v},t) + \frac{Ze}{m}\big(\boldsymbol{E}(\boldsymbol{x},\boldsymbol{t}) + \boldsymbol{v} \times \boldsymbol{B}(\boldsymbol{x},\boldsymbol{t})\big) \cdot \frac{\partial f}{\partial \boldsymbol{v}}(\boldsymbol{x},\boldsymbol{v},t) = 0$$

and equilibrium in the absence of the wave

$$\boldsymbol{v} \cdot \frac{\partial F_0}{\partial \boldsymbol{x}}(\boldsymbol{x},\boldsymbol{v},t) + \frac{Ze}{m}\big(\boldsymbol{E}_0(\boldsymbol{x}) + \boldsymbol{v} \times \boldsymbol{B}_0(\boldsymbol{x})\big) \cdot \frac{\partial F_0}{\partial \boldsymbol{v}}(\boldsymbol{x},\boldsymbol{v},t) = 0$$

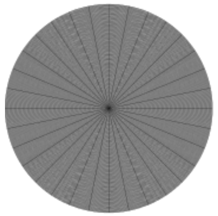
Letting $f(\boldsymbol{x},\boldsymbol{v},t) = F_0(\boldsymbol{x},\boldsymbol{v}) + \delta f(\boldsymbol{x},\boldsymbol{v},t)$, we seek an evolution equation for the unknown perturbed distribution δf, arriving at

$$\frac{\partial \delta f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial \delta f}{\partial \boldsymbol{x}} + \frac{Ze}{m}\big(\boldsymbol{E}_0 + \boldsymbol{\delta E} + \boldsymbol{v} \times (\boldsymbol{B}_0 + \boldsymbol{\delta B})\big) \frac{\partial \delta f}{\partial \boldsymbol{v}} = -\frac{Ze}{m}(\boldsymbol{v} \times \delta \boldsymbol{B} + \delta \boldsymbol{E}) \frac{\partial F_0}{\partial \boldsymbol{v}}$$

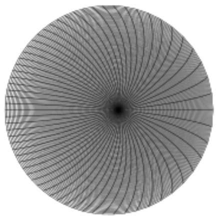
HALO

(HAgis LOcust)

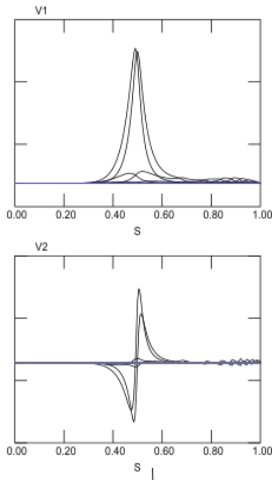
EFIT



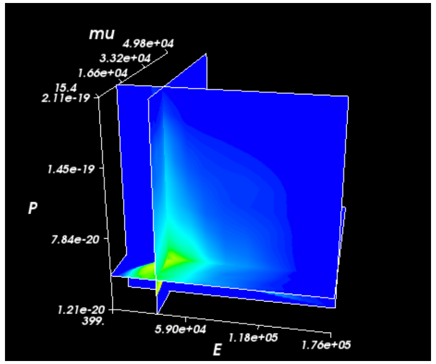
HELENA



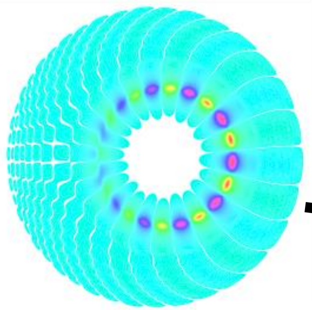
MISHKA



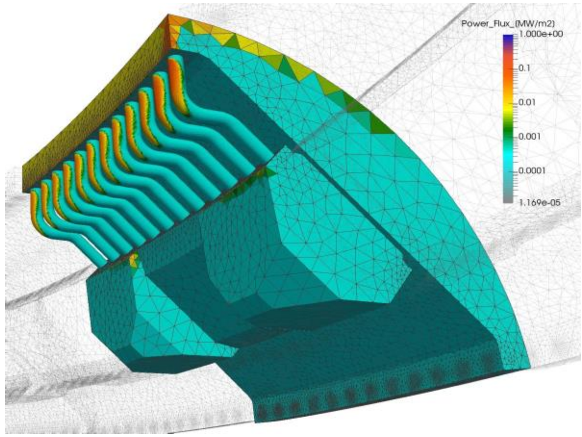
Equilibrium distribution F_0 of fast ions:
Alphas
Beam
ICRH minority



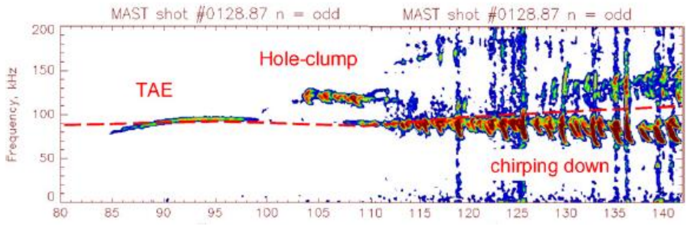
HALO



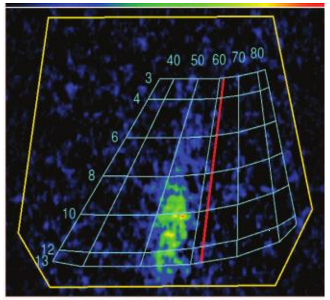
LOCUST-GPU



Fokker-Plank codes:
LOCUST-GPU
TRANSP
TORIC
ASCOT
….



Gryaznevich, M. P., & Sharapov, S. E. (2006)

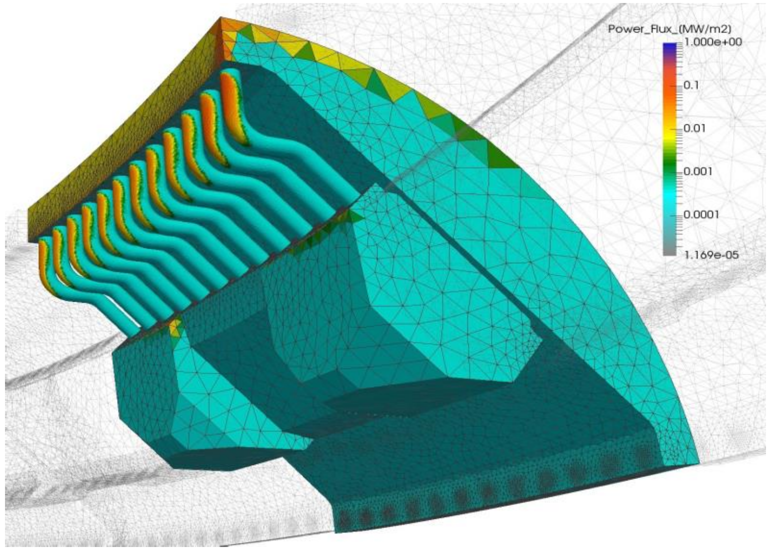


Kiptily, V. G. et al. 2018

Discrete $\delta \boldsymbol{J}_{\boldsymbol{fast}}$ - LOCUST-GPU

- Full-orbit
- Tracks particles inside and outside the separatrix
- Supports detailed 3D wall models for calculation of power loadings to plasma facing components
- GPU allows cheap high statistics (millions of particles if required)

$$L(\boldsymbol{x}, \boldsymbol{v}, \dot{\boldsymbol{x}}, \dot{\boldsymbol{v}}, t) = (m\boldsymbol{v} + e\boldsymbol{A}_0 + e\delta\boldsymbol{A}) \cdot \dot{\boldsymbol{x}} - \left(e\Phi_0 + e\delta\Phi + \frac{m}{2}\boldsymbol{v}\cdot\boldsymbol{v}\right)$$

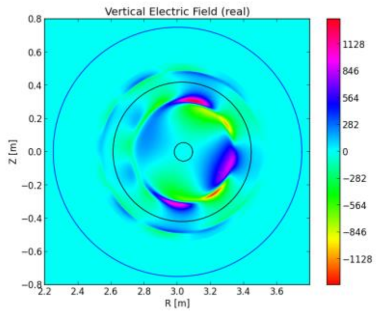


HALO improvements on drift HAGIS code

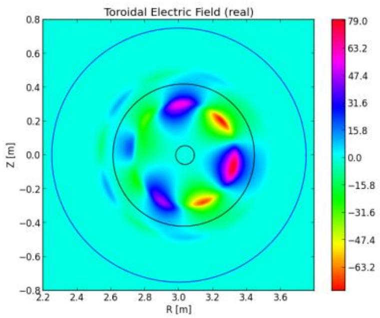
- Can look at CAEs and low frequency eigenmodes
- Track particles anywhere in vessel
- Multiple species
- Power transfer depends on FLR effects as $J_0(k_\perp \rho)$

$$\frac{d}{dt}\delta f_i(t) = -\frac{Ze}{m}\left(\boldsymbol{v_i}(t) \times \delta\boldsymbol{B}(\boldsymbol{x_i}(t), t) + \delta\boldsymbol{E}(\boldsymbol{x_i}(t), t)\right) \cdot \left(\frac{\partial F_0}{\partial \boldsymbol{v}}\right)_{\boldsymbol{x}} (\boldsymbol{x_i}(t), \boldsymbol{v_i}(t))$$

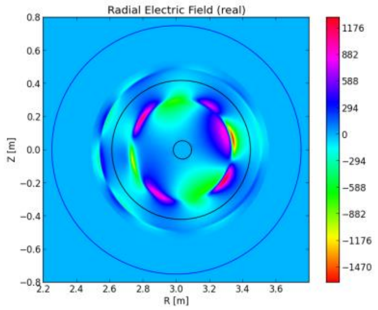
Discrete unperturbed eigenmodes



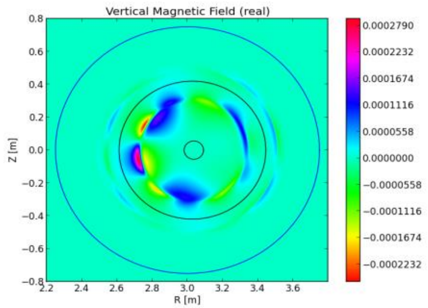
$\delta E_R(R,\phi,Z)$



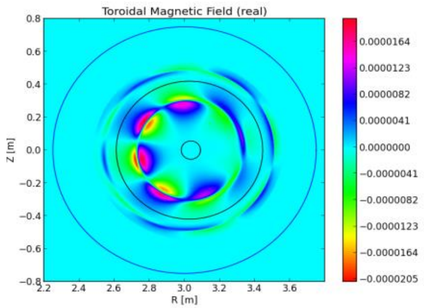
$\delta E_\phi(R,\phi,Z)$



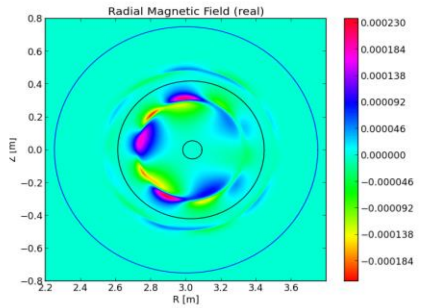
$\delta E_Z(R,\phi,Z)$



$\delta B_R(R,\phi,Z)$



$\delta B_\phi(R,\phi,Z)$



$\delta B_Z(R,\phi,Z)$

HELENA

straight field

(s,θ,ζ)

MISHKA

$$J\delta B(s,\theta) = \sum_m \overline{JB}(s,m)e^{im\theta}$$

$$\delta E(s,\theta) = \sum_m \bar{E}(s,m)e^{im\theta}$$

HALO

cylindrical

$\delta \boldsymbol{B}(R,\phi,Z)$
$\delta \boldsymbol{E}(R,\phi,Z)$

Benchmark: test particles follow the correct orbit when the fields are large

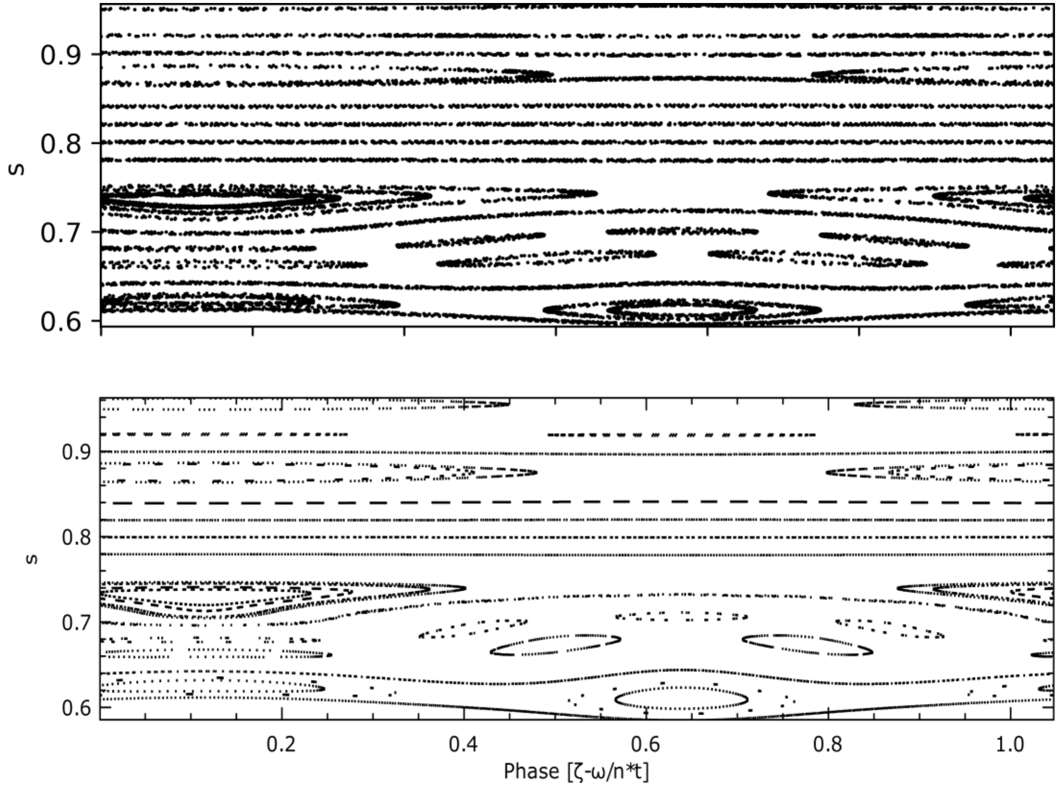
$$\frac{dB_r}{B_0} = 3 \times 10^{-3}.$$

Test particles launched at the Alfven speed get trapped in the waves

$$v_\parallel = v_A$$

HALO

HAGIS



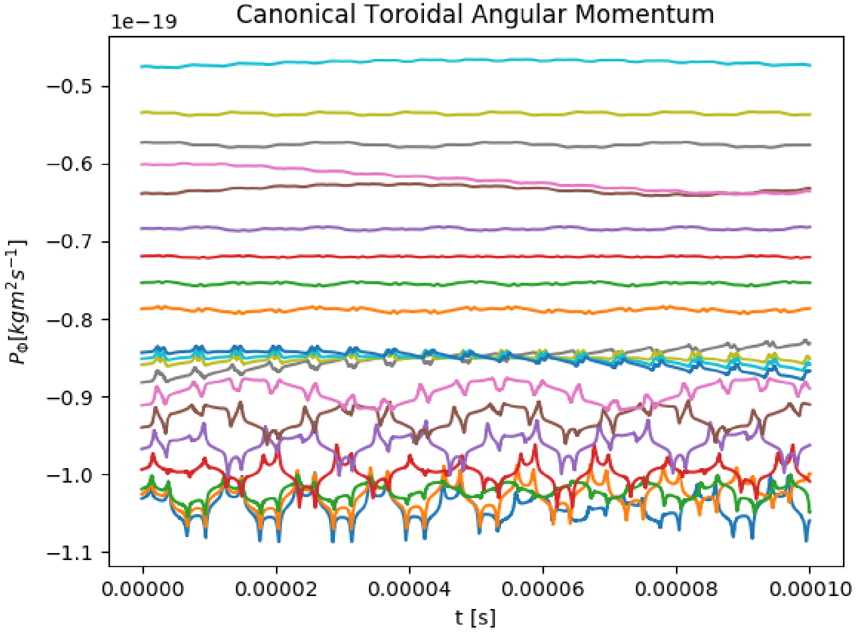
Benchmark: resonant particles obey conservation laws when the fields are large

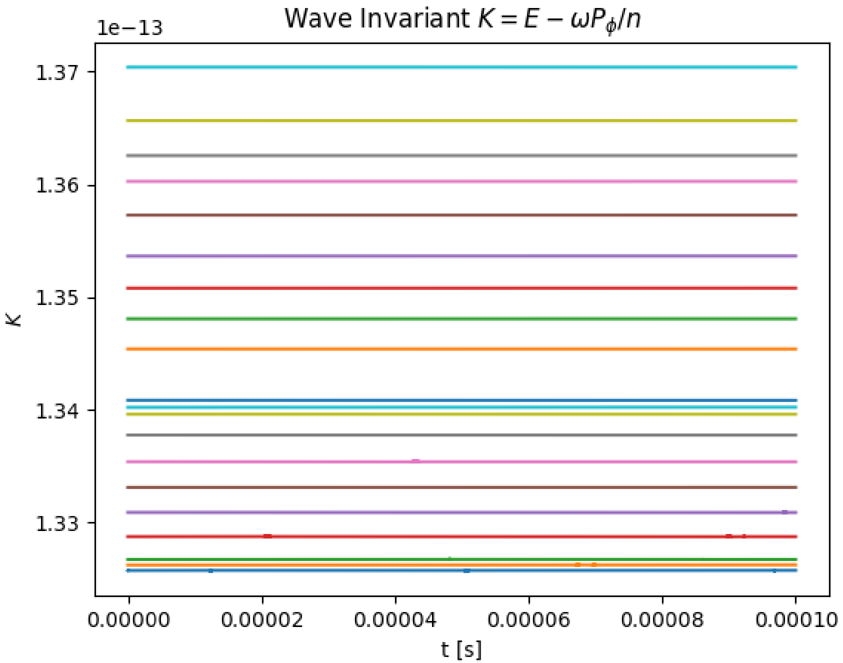
$$K = E - \frac{\omega}{n} P_\phi$$

$$L(\boldsymbol{x}, \boldsymbol{v}, \dot{\boldsymbol{x}}, \dot{\boldsymbol{v}}, t) = (m\boldsymbol{v} + e\boldsymbol{A}_0 + e\boldsymbol{\delta A}) \cdot \dot{\boldsymbol{x}} - \left(e\Phi_0 + e\delta\Phi + \frac{m}{2}\boldsymbol{v} \cdot \boldsymbol{v}\right)$$

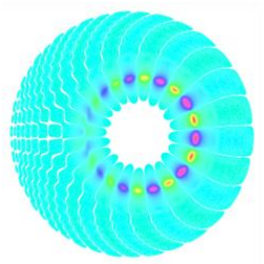
$$\frac{d}{dt} E = -\frac{\partial L}{\partial t}$$

$$\frac{d}{dt} P_\phi = \frac{\partial L}{\partial \phi}$$





Benchmark: linear stability and reduced FLR effects in TAE drive



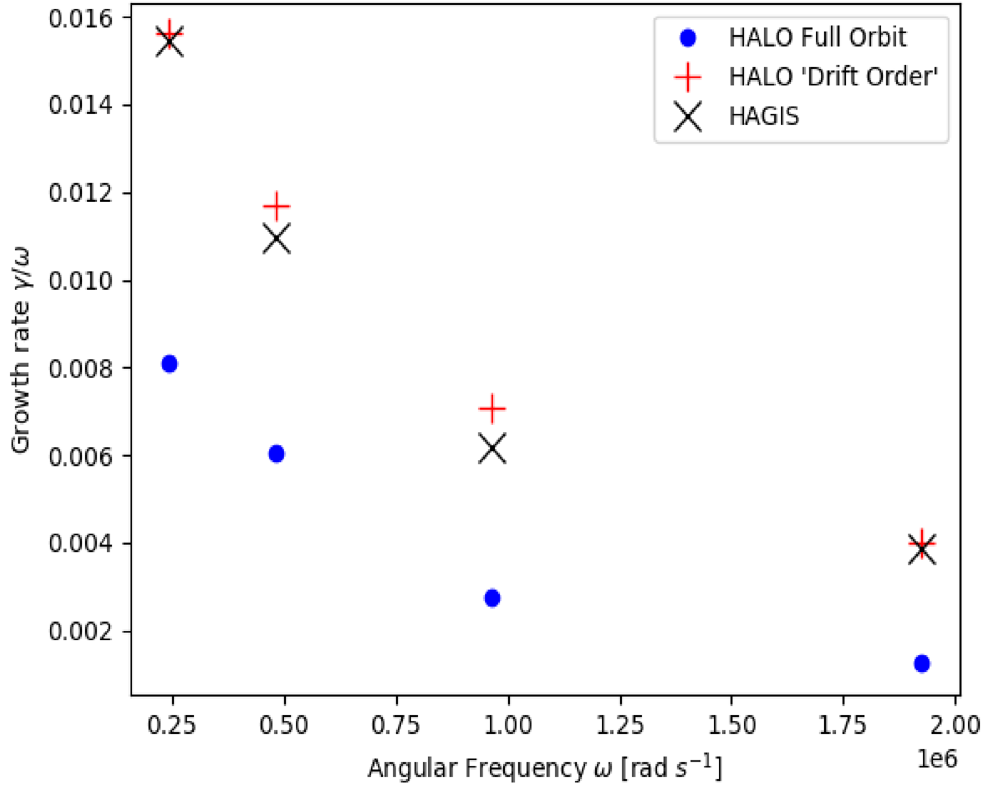
$n = 6$ TAE

Parameter	Value
$\epsilon = a/R_0$	0.25
$v_A/2a\Omega_c$	0.03
R_0	3.0
B_0	3.0
D:T (%)	50:50
q_0	1.82
q_{95}	3.31
T_e	20keV
T_i	20keV
$n_\alpha(0)/n_e(0)$	1%

$$\boldsymbol{V} \equiv \dot{\boldsymbol{X}}, \boldsymbol{x} \equiv \boldsymbol{X} + \boldsymbol{\rho}$$

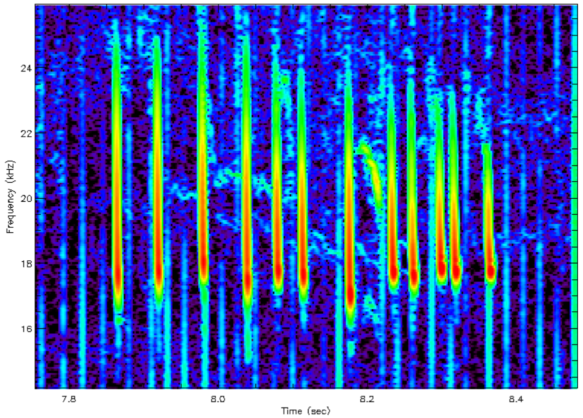
HALO mimics HAGIS when
$\delta\boldsymbol{E}\big(\boldsymbol{x_i}(\boldsymbol{t}); \omega_j\big) \rightarrow \delta\boldsymbol{E}\big(\boldsymbol{X_i}(t); \omega_j\big)$

FLR effect should be $J_0(k_\perp \rho)$

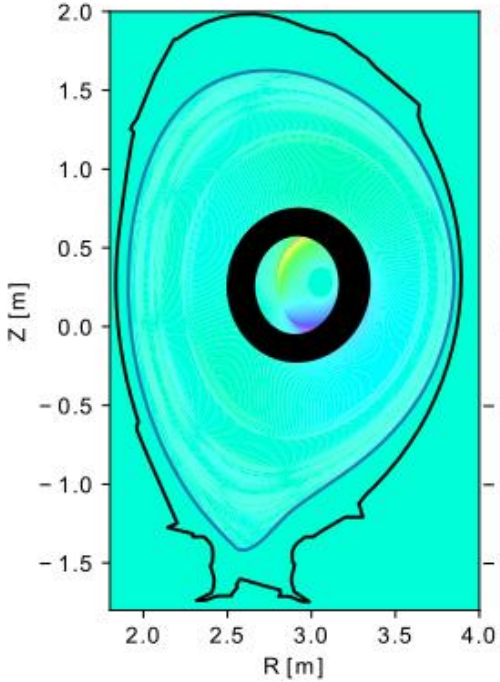


A simple calculation shows that such a decrease in drive is to be expected for the benchmark case; $k_\perp \approx \frac{m}{r} \approx \frac{12.5}{0.4}$, and at the Alfven speed $\rho \approx 0.045m$ giving $J_0(k_\perp \rho)$ =0.56.

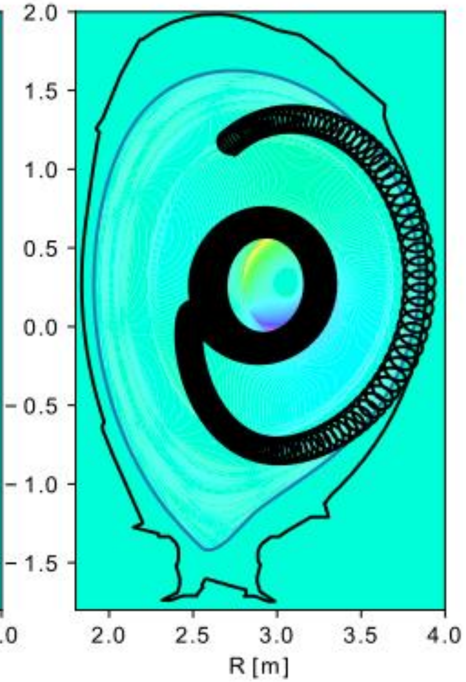
Interpretive studies that don’t compute the wave evolution: Sawbone losses on JET



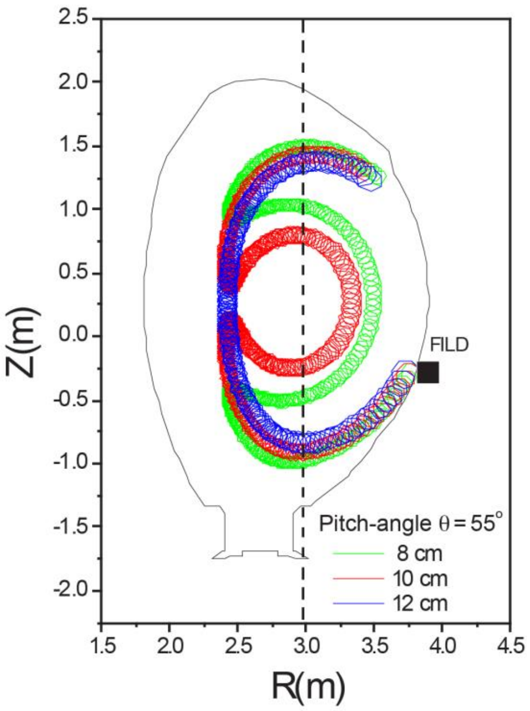
HALO without Mode

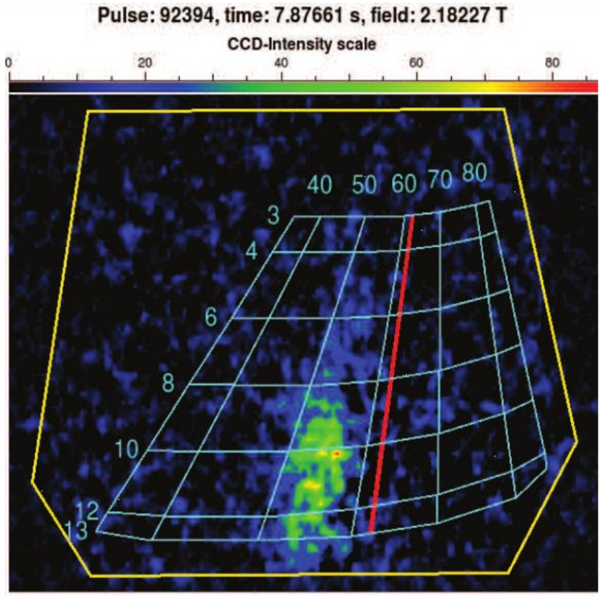


HALO with Mode



Backtracking from FILD





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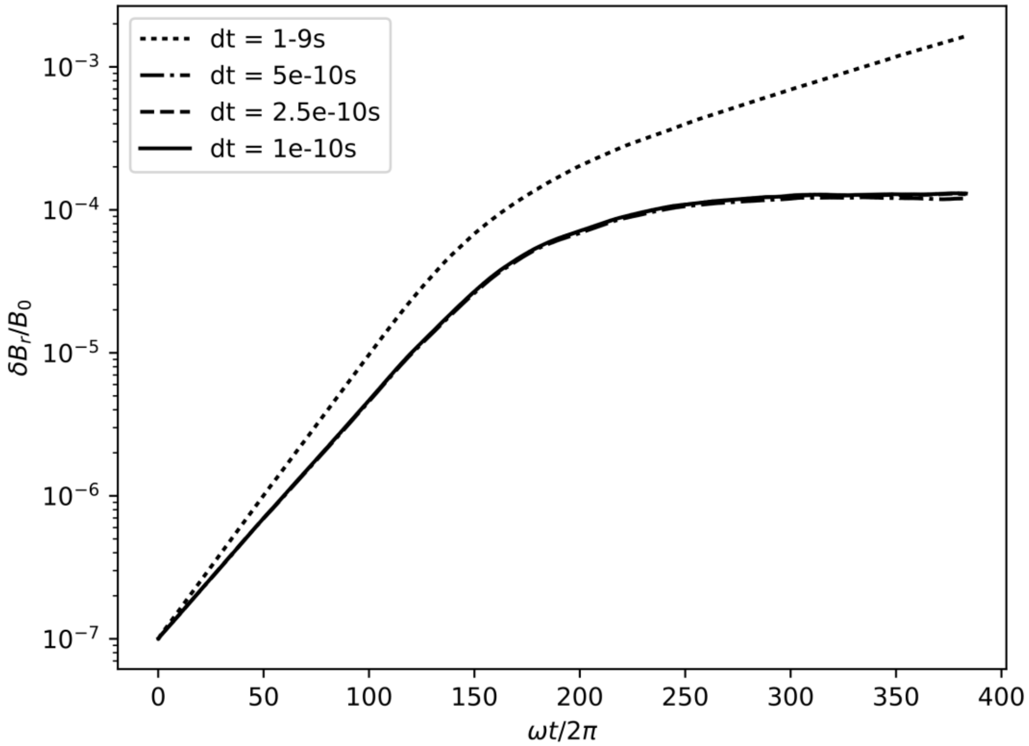
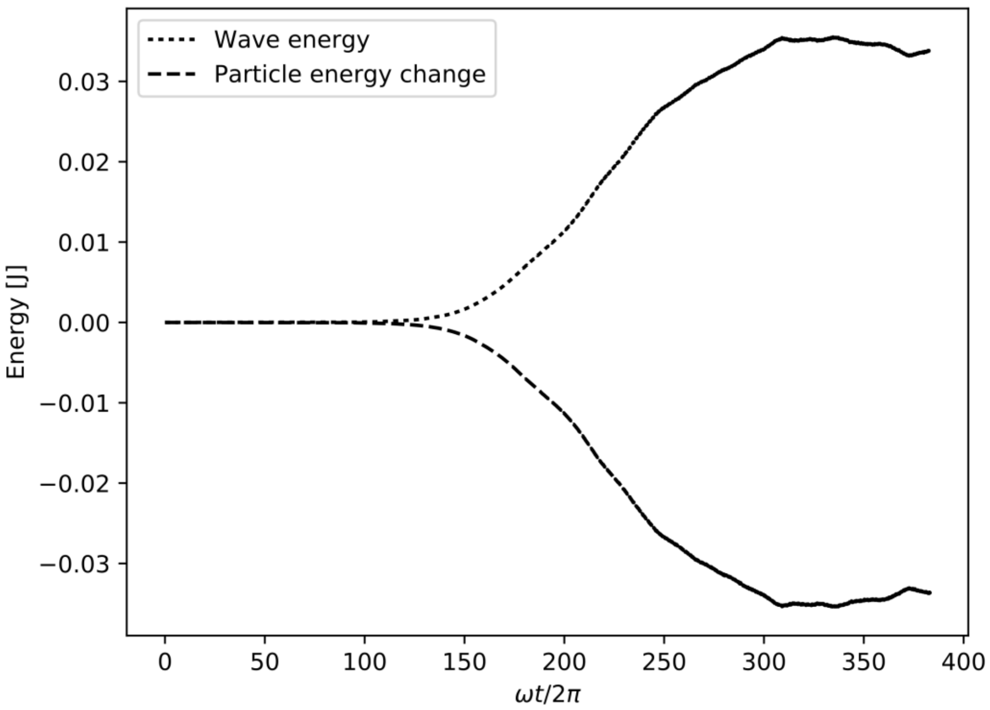
Nonlinear test: growth and saturation





Nonlinear test: BGK waves of holes and clumps

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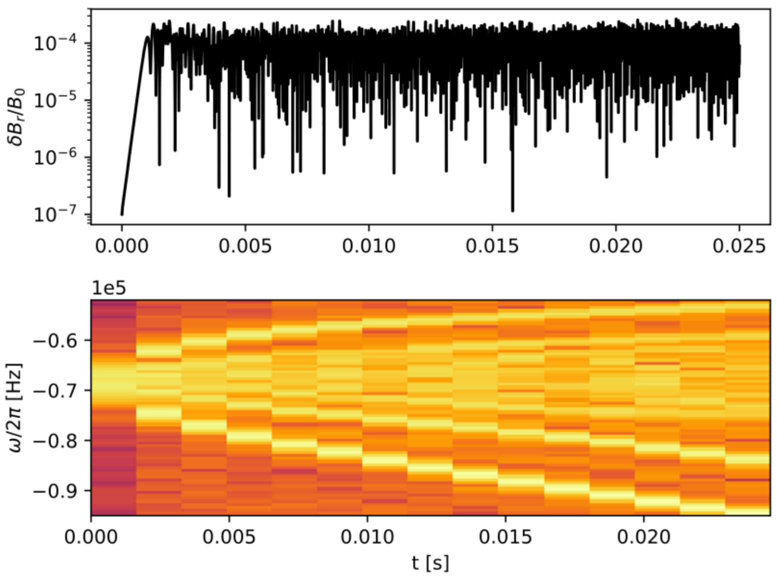
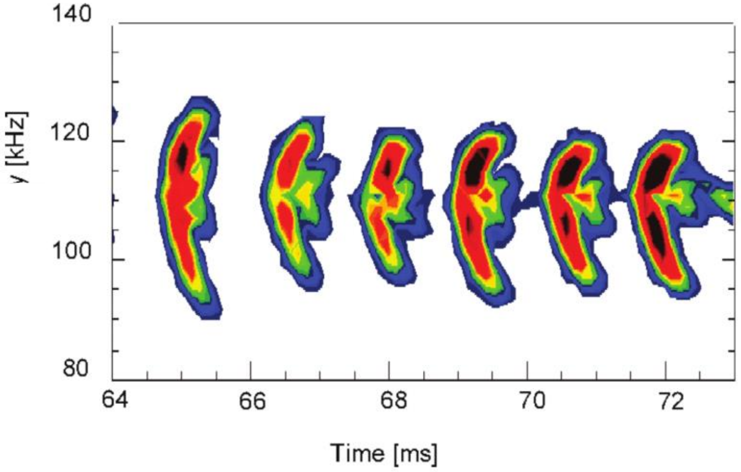
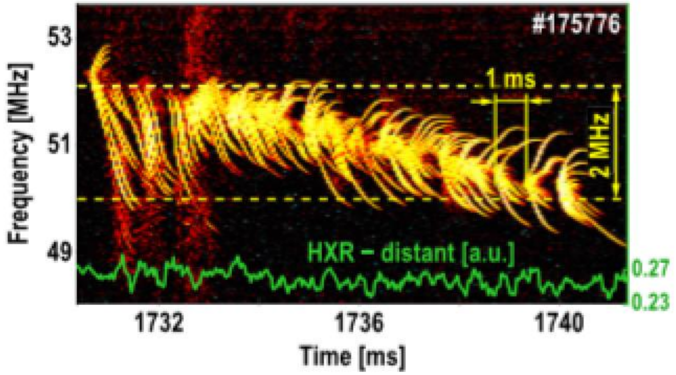
HALO benchmark case

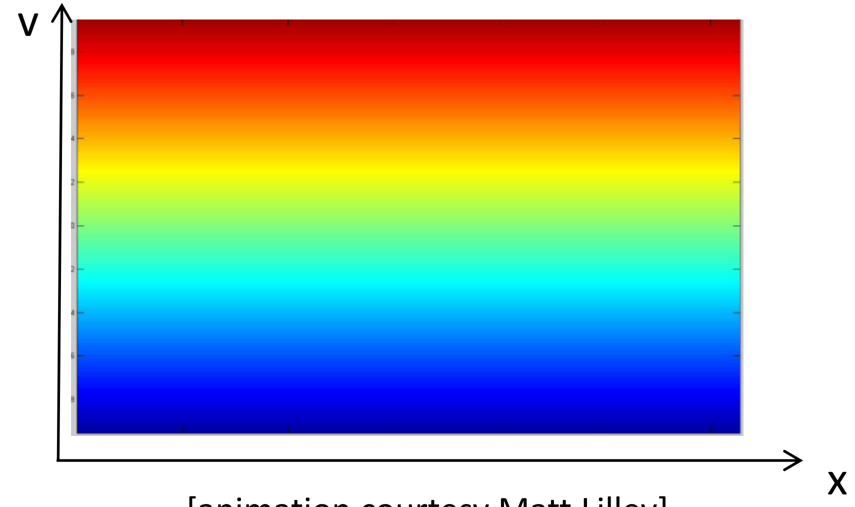


MAST TAEs

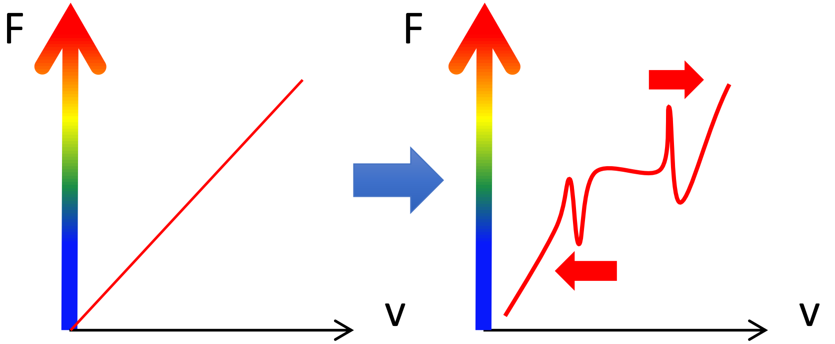


DIIID compressional modes





[animation courtesy Matt Lilley]



Berk, H., Breizman, B. et al. (1997). Physics Letters A, 234(3), 213–218. doi:10.1016/S0375-9601(97)00523-9
Pinches, S. D. et al.(2004). Plasma Physics and Controlled Fusion, 46(7), S47–S57. doi:10.1088/0741-3335/46/7/S04
A. Lvovskiy et al 2019 Nucl. Fusion 59 124004

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Linear vs nonlinear code operation

nonlinear

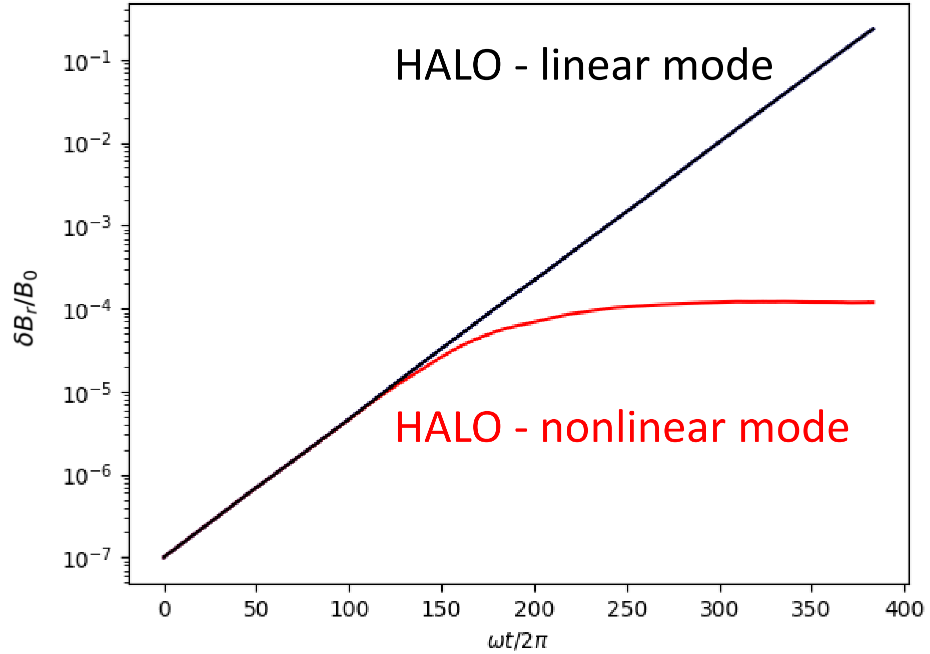
$$\frac{\partial \delta f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial \delta f}{\partial \boldsymbol{x}} + \frac{Ze}{m}\left(\boldsymbol{E}_0 + \boldsymbol{\delta E} + \boldsymbol{v} \times (\boldsymbol{B}_0 + \boldsymbol{\delta B})\right)\frac{\partial \delta f}{\partial \boldsymbol{v}} = -\frac{Ze}{m}(\boldsymbol{v} \times \delta\boldsymbol{B} + \delta\boldsymbol{E})\frac{\partial F_0}{\partial \boldsymbol{v}}$$

$$\left.\frac{\mathrm{d}\delta f}{dt}\right|_{\text{perturbed orbit}} = -\frac{Ze}{m}(\boldsymbol{v} \times \delta\boldsymbol{B} + \delta\boldsymbol{E})\frac{\partial F_0}{\partial \boldsymbol{v}}$$

linear

$$\frac{\partial \delta f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial \delta f}{\partial \boldsymbol{x}} + \frac{Ze}{m}(\boldsymbol{E}_0 + \boldsymbol{v} \times \boldsymbol{B}_0)\frac{\partial \delta f}{\partial \boldsymbol{v}} = -\frac{Ze}{m}(\boldsymbol{v} \times \delta\boldsymbol{B} + \delta\boldsymbol{E})\frac{\partial F_0}{\partial \boldsymbol{v}}$$

$$\left.\frac{\mathrm{d}\delta f}{dt}\right|_{\text{unperturbed orbit}} = -\frac{Ze}{m}(\boldsymbol{v} \times \delta\boldsymbol{B} + \delta\boldsymbol{E})\frac{\partial F_0}{\partial \boldsymbol{v}}$$



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Generalization of method to any wave and any Hamiltonian motion

$$\delta \boldsymbol{J}_{\boldsymbol{i},\boldsymbol{fast}}(\boldsymbol{q},t) = Ze\int d\boldsymbol{p}\,\hat{f}(\boldsymbol{q},\boldsymbol{p},t)\boldsymbol{v}(\boldsymbol{q},\boldsymbol{p},t)$$

$$\dot{\boldsymbol{q}} = \frac{\partial H(\boldsymbol{q},\boldsymbol{p},t)}{\partial \boldsymbol{p}}, \dot{\boldsymbol{p}} = -\frac{\partial H(\boldsymbol{q},\boldsymbol{p},t)}{\partial \boldsymbol{q}} \qquad \frac{\partial \hat{f}(\boldsymbol{q},\boldsymbol{p},t)}{\partial t} + \frac{\partial}{\partial \boldsymbol{q}}\cdot\left(\dot{\boldsymbol{q}}\hat{f}(\boldsymbol{x},\boldsymbol{p},t)\right) + \frac{\partial}{\partial \boldsymbol{p}}\cdot\left(\dot{\boldsymbol{p}}\hat{f}(\boldsymbol{x},\boldsymbol{p},t)\right) = 0$$

The distribution function is separated into equilibrium and perturbed portions $\hat{f}(\boldsymbol{q},\boldsymbol{p},t) = \widehat{F_0}(\boldsymbol{q},\boldsymbol{p}) + \delta\hat{f}(\boldsymbol{q},\boldsymbol{p},t)$, as is the Hamiltonian $H(\boldsymbol{q},\boldsymbol{p},t) = H_0(\boldsymbol{q},\boldsymbol{p}) + \delta H(\boldsymbol{q},\boldsymbol{p},t)$ with the equilibrium satisfying

$$\frac{\partial H_0(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{p}}\cdot\frac{\partial \widehat{F_0}(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{q}} - \frac{\partial H_0(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{q}}\cdot\frac{\partial \widehat{F_0}(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{p}} = 0$$

$$\left.\frac{\mathrm{d}\delta\hat{f}}{dt}\right|_{\text{perturbed orbit}} = -\frac{\partial \delta H(\boldsymbol{q},\boldsymbol{p},t)}{\partial \boldsymbol{p}}\cdot\frac{\partial \widehat{F_0}(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{q}} + \frac{\partial \delta H(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{q}}\cdot\frac{\partial \widehat{F_0}(\boldsymbol{q},\boldsymbol{p})}{\partial \boldsymbol{p}}$$

Generalization to any discrete eigenmode

$$\delta W_{TAE} = \frac{1}{2\mu_0}\int dx\frac{\boldsymbol{e}^{\dagger}(\boldsymbol{x};\omega_j)\boldsymbol{e}(\boldsymbol{x};\omega_j)}{v_A^2(\boldsymbol{x})} = \frac{1}{\mu_0}\int dx\frac{\overline{\delta\boldsymbol{e}^2}(\boldsymbol{x};\omega_j)}{v_A^2(\boldsymbol{x})} = \frac{1}{\mu_0}\int dx\ \overline{\delta\boldsymbol{b}^2}(\boldsymbol{x};\omega_j)$$

$$\delta W_{COLD} = -\frac{i}{4}\int d\boldsymbol{x}d\boldsymbol{x}'\ \boldsymbol{e}^{\dagger}(\boldsymbol{x};\omega_j)\frac{\partial \mathrm{g}_{\mathrm{COLD}}(\boldsymbol{x},\boldsymbol{x}',\omega_j)}{\partial\omega}\ \boldsymbol{e}(\boldsymbol{x}';\omega_j)$$

An cold plasma wave below the electron cyclotron frequency

$$\frac{\partial\sigma_{COLD}}{\partial\omega}(\boldsymbol{x},\boldsymbol{x}',\omega) = -\delta(x-\boldsymbol{x}')\begin{pmatrix} i\frac{1}{v_A^2}\frac{\Omega_c^2(\Omega_c^2+\omega^2)}{(\Omega_c^2-\omega^2)^2} & -\frac{2\Omega_c^2\omega}{(\Omega_c^2-\omega^2)^2}\frac{ne}{B_0} & 0 \\ \frac{2\Omega_c^2\omega}{(\Omega_c^2-\omega^2)^2}\frac{ne}{B_0} & i\frac{1}{v_A^2}\frac{\Omega_c^2(\Omega_c^2+\omega^2)}{(\Omega_c^2-\omega^2)^2} & 0 \\ 0 & 0 & i\left(\frac{n_iZ^2e^2}{m_i\omega^2}+\frac{ne^2}{m_e\omega^2}\right)\end{pmatrix}$$

Conclusion and further work

- The new HALO code adopts the same wave physics approach of HAGIS, but takes advantage of the power of LOCUST-GPU to describe the full motion of fast particles
- The code reproduces all the relevant physics in HAGIS and has already demonstrated the importance of FLR effects for JET-like conditions with high-q
- Coming applications of existing code
  - Multispecies modelling for alpha driven TAEs on JET (NBI, ICRH, alphas)
  - ITER cases with high toroidal mode number for FLR effects
  - TAE studies on MAST
- Tackle compressional modes and low frequency modes in the coming year, extending beyond MHD modes