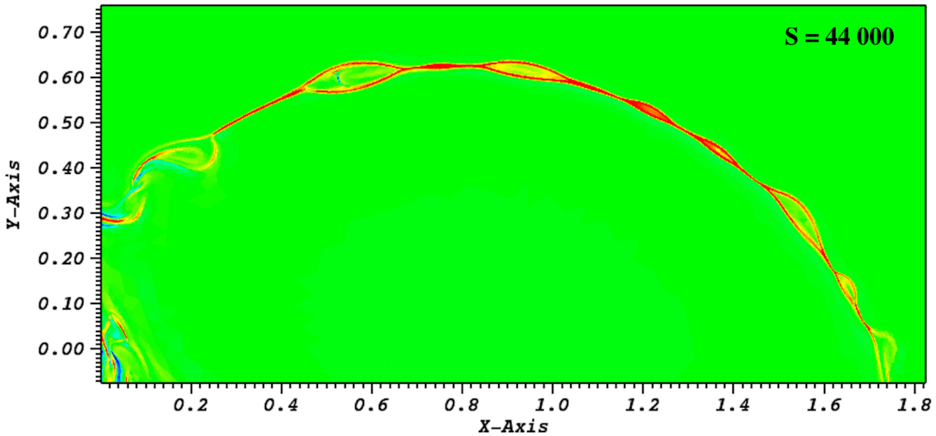
Tilt Instability and Formation of Plasmoid Chains



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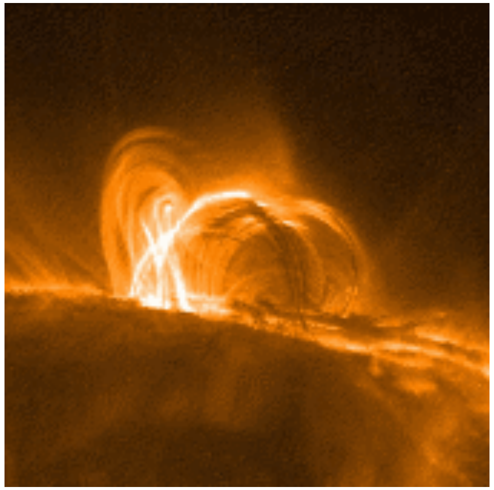
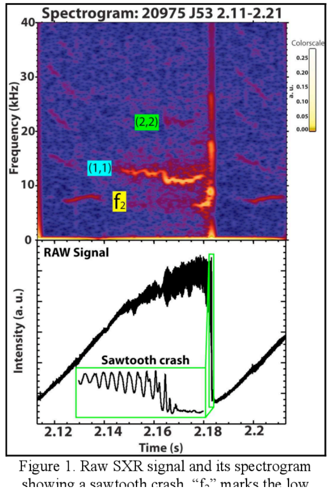
Motivation: solar flares & tokamak disruptions

- **Eruptive events in strongly magnetized plasmas => fast magnetic reconnection !**

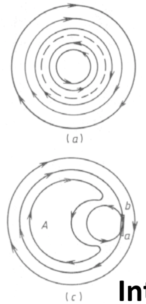
Solar flare in the solar corona (in UV by TRACE satellite)



Internal disruption in tokamaks (ASDEX-upgrade)



Papp et al. 2009



Internal kink mode => sawtooth crash

Flare: sudden and fast brightening (few minutes)

Magnetic energy released ~ 10^{25} joules in (60000 km)^3
- **-heating**
- **-plasma acceleration (blobs, particles)**

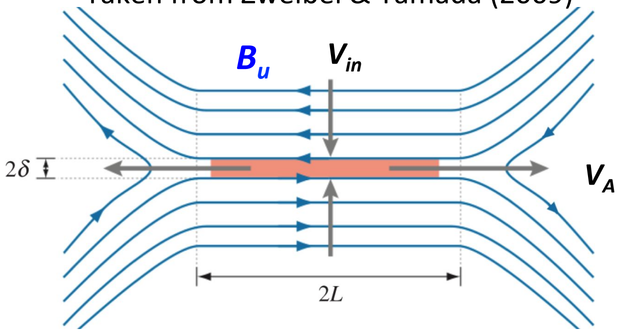
Disruption: fast (100 μs) magnetic topology reorganisation

=> Alfvénic time scale

Models for fast magnetic reconnection (MHD)

- **Sweet-Parker (SP) and Kadomtsev models**

Taken from Zweibel & Yamada (2009)



Central (resistive) current layer

$$J_{max} = S^{1/2} B_u / L$$

$$L/\delta = S^{1/2}$$

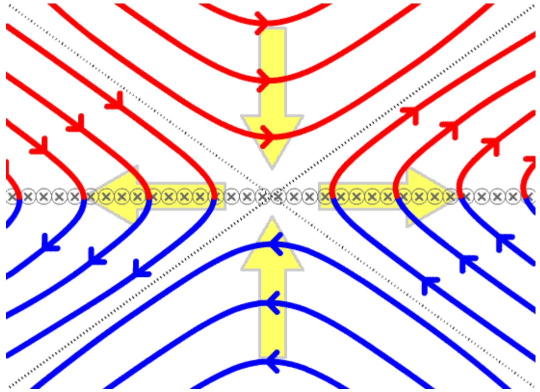
(L and B_u fixed)

Reconnection rate solution is:

$$V_{in}/V_A = S^{-1/2}$$

Sweet (1958) & Parker (1957)
Kadomstev (1974)
Priest & Forbes (1986)

Schematic view (Wikipédia)



2D incompresible MHD

Stationary form: change of the magnetic topology around an X-point

-inflow plasma speed => process rate !
-outflow plasma speed = Alfvén speed

V_{in} : inflow velocity
V_A : Alfvén speed (with upstream reversal field component B_u)
$S = LV_A/\eta$ (Lundquist number)
η : plasma resistivity (magnetic diffusivity)
(viscosity is neglected)

Models for fast magnetic reconnection (MHD)

- **Sweet-Parker (SP) and Kadomtsev models**

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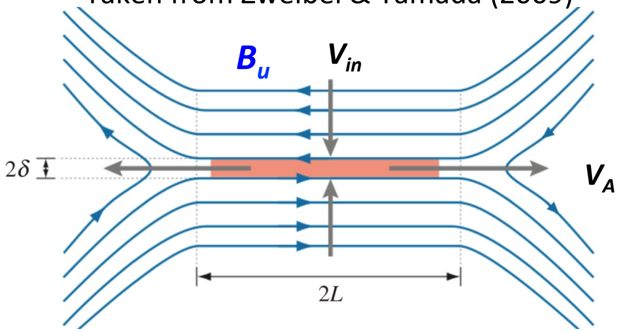

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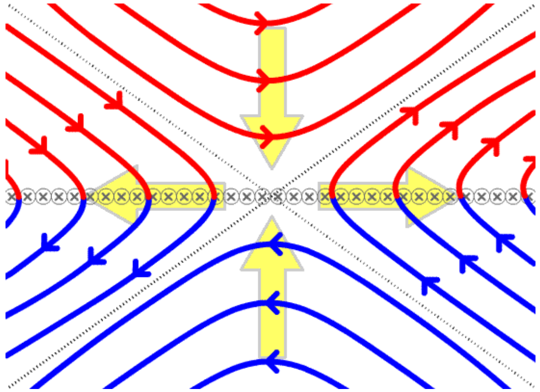
V_{in} : inflow velocity
V_A : Alfvén speed (with upstream reversal field comp.)
$S = LV_A/\eta$ (Lundquist number)
η : plasma resistivity (magnetic diffusivity)

$S = S^* (1 - q_0)\, B_\theta/B$ (kink in tokamak)

Schematic view (Wikipédia)



2D incompresible MHD

Stationary form: change of the magnetic topology around an X-point

-inflow plasma speed => process rate !
-outflow plasma speed = Alfvén speed

- SP rates of order 10^{-3} and 10^{-5} are predicted as $S \sim 10^{5\text{-}6}$ (tokamak) and $10^{10\text{-}12}$ (solar corona)
- but rates of order $10^{-2} - 10^{-1}$ are required

=> <u>SP process is too slow !</u>

Models for fast magnetic reconnection (MHD)

- Plasmoids formation – SP current sheet ($L/\delta \approx S^{1/2}$) is unstable when $S > S_c$ ($S_c \sim 10^4$)

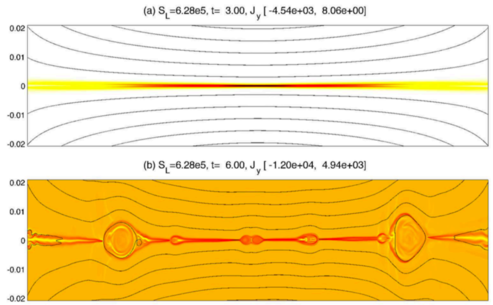
- Linear theory – Loureiro et al. 2007 => $\boxed{\text{growth rate} \quad \gamma_p L/V_A \sim S^{1/4}, \text{ and } k_p L \sim S^{3/8}}$

- Viscosity effect ($P_m = \nu/\eta$) - Comisso & Grasso 2016 => $\gamma_p L/V_A \sim S^{1/4} (1+P_m)^{-5/8}$ and $k_p L \sim S^{3/8} (1+P_m)^{-3/16}$

- Confirmed by MHD numerical simulations using static/stationary SP current layers as an initial set-up – Samtaney et al. 2009

+ non linear evolution => $\boxed{\text{New regime of reconnection with an accelerated rate} \sim 10^{-2} \text{ independent of } S}$

Plasmoids = Tearing-like modes



Bhattacharjee et al. 2009

Models for fast magnetic reconnection (MHD)

- Plasmoids formation – SP current sheet ($L/\delta \approx S^{1/2}$) is unstable when $S > S_c$ ($S_c \sim 10^4$)

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at t_1 $\qquad$ $t_2 > t_1$ $\qquad$ $t_3 > t_2 > t_1$

1) Controversy arose on the plasmoids growth (γ_p -> ∞ as S -> ∞ contradicts frozen-in law)

-> Pucci & Velli (2014): currents sheets with $L/\delta \approx S^{1/3}$ => $\boxed{\text{linear growth rate} \;\; \gamma_p L/V_A \approx 1}$

-> Comisso et al. (2016): dynamically forming SP sheets => <u>non simple power laws</u> for γ_p, k_p (dominant mode => least time), γ_p <u>can be super-Alfvénic</u> but remains finite for infinite S, and previous power laws are valid for S -> S_c

2) Non linear stage in simulations => the reco. rate of ~ 10^{-2} independent of S, is a time-averaged value during a time-dependent stochastic reconnection regime, it but is not clearly explained !

→ <u>The aim of this talk !</u>

FINMHD code

- **Choice of MHD model (2D incompressible) –> *J* - ω formalism**

Reduced MHD equations (current density *J* – vorticity ω) -> 1st time used for reconnection/plasmoids

$$\left[\begin{array}{l} \dfrac{\partial \omega}{\partial t} + (\vec{V}.\vec{\nabla})\omega = (\vec{B}.\vec{\nabla})J + \nu \nabla^2 \omega \\ \dfrac{\partial J}{\partial t} + (\vec{V}.\vec{\nabla})J = (\vec{B}.\vec{\nabla})\omega + F(\phi,\psi) + \eta \nabla^2 J \\ \nabla^2 \phi = -\omega \\ \nabla^2 \psi = -J \end{array}\right]$$

viscosity

resistivity

$$F(\phi,\psi) = 2\frac{\partial^2 \phi}{\partial x \partial y}\left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}\right) - 2\frac{\partial^2 \psi}{\partial x \partial y}\left(\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2}\right)$$

See also Philip et al. 2006, Lankalappali et al. 2007

xOy

Simulation plane

$$\vec{B} = \vec{\nabla} \times (\psi \vec{e}_z)$$

$$(B_x, B_y) = \left(\frac{\partial \psi}{\partial y}, -\frac{\partial \psi}{\partial x}\right)$$

$$\vec{V} = \vec{\nabla} \times (\phi \vec{e}_z)$$

$$(V_x, V_y) = \left(\frac{\partial \phi}{\partial y}, -\frac{\partial \phi}{\partial x}\right)$$

$$\vec{e}_z J = \vec{\nabla} \times \vec{B}$$

$$\vec{e}_z \omega = \vec{\nabla} \times \vec{V}$$

Definitions

-> zero divergence of magnetic field and velocity field are ensured

-> nearly symmetric form

-> maximum spatial derivative is 2nd order (standard $\psi - \omega$ model is 3rd order)

FINMHD code

- ***J* - *ω* formalism : space/time discretization**

$$\left[\begin{array}{l} \dfrac{\omega^{n+1} - \omega^{n} \circ X^{n}}{\Delta t} - (\vec{B}^{n}.\vec{\nabla})J^{n+1} - \nu\nabla^{2}\omega^{n+1} = 0 \\ \dfrac{J^{n+1} - J^{n} \circ X^{n}}{\Delta t} - (\vec{B}^{n}.\vec{\nabla})\omega^{n+1} - \eta\nabla^{2}J^{n+1} = F^{n}(\phi,\psi) \end{array}\right]$$

$$\nabla^{2}\phi^{n+1} = -\omega^{n+1}$$

$$\nabla^{2}\psi^{n+1} = -J^{n+1}$$

-> **A finite-element discretization**:
Lagrange second order – triangular P_2
$\Rightarrow$ **Characteristic Galerkin method**

-> **A semi-implicit scheme**:
(1st order and 2nd order predictor-corrector versions with adaptive time step are developed)
<- n means at t^n

$$\frac{D\omega}{Dt} = \left(\frac{\partial}{\partial t} + \vec{V}.\vec{\nabla}\right)\omega \rightarrow \frac{\omega^{n+1} - \omega^{n} \circ X^{n}}{\Delta t}$$

for Lagrangian derivative
=> Method of characteristics
(Pironneau method 1988)

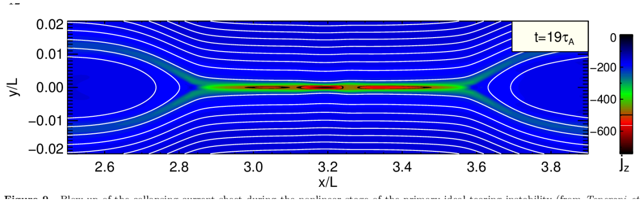
Code optimization:
internship of I. Moufid (2018)

Freefem++ software (see https://freefem.org/, F. Hecht & coll.):
-matrix (stiffness/mass) elements are automatically assembled
-large choice of (direct and iterative) linear solvers
-efficient spatial adaptivity method (Hessian of *J*) -> non structured adaptive mesh !

=> FINMHD code – see Baty 2019 in ApJS 243, 23

Which setup ?

- **Ideally stable and resistively unstable Harris-type current layer**

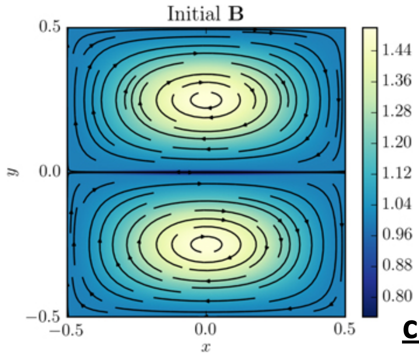


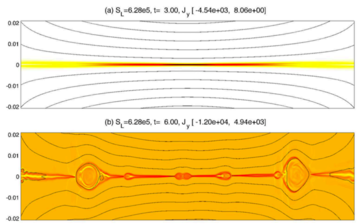
-> used by Velli and coll.

(codes with at least one periodicity in general)

probably not the best setup for our aim !

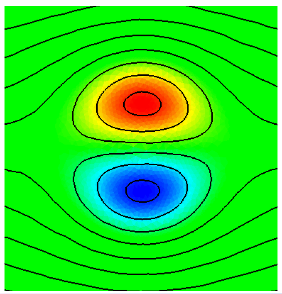
- **Ideally unstable configuration (current-driven mode)**





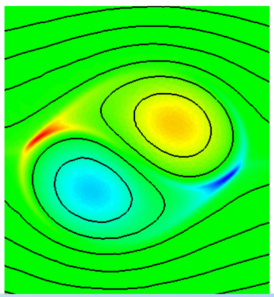
coalescence

-> used by Bhattacharjee and coll.
(the current layer is in direct contact with the numerical boundary + initial arbitrary layer)
See Huang et al. 2017 -> similar study !



Colored contour map of current density

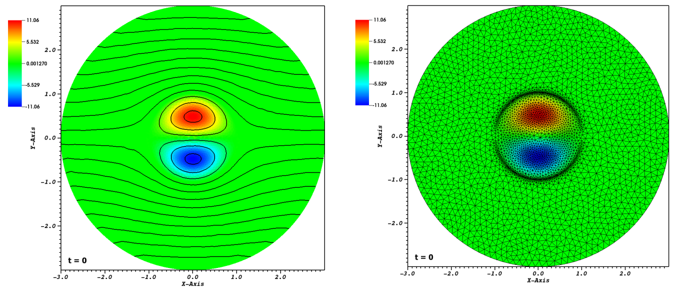
tilt



-> never used to study magnetic reconnection except by Keppens et al. 2014
(numerical boundary can be chosen far and two twin current sheets are self-consistently formed)

Results

- **Magnetic reconnection with plasmoids during Tilt instability - <u>SP regime</u>**



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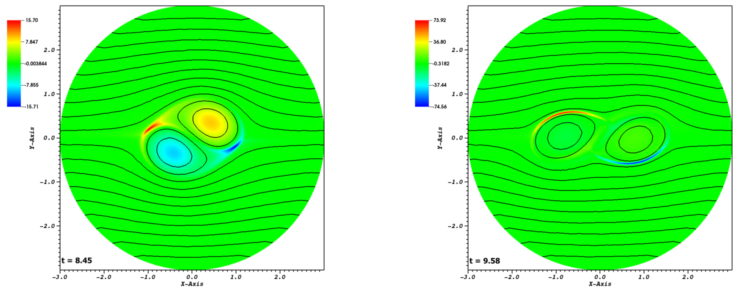
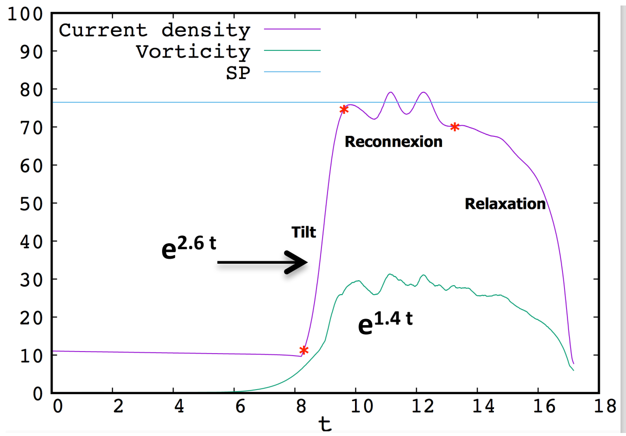


Initial setup (dipole vortex – see Richard et al. 1990)
Circular boundary is numerically advantagous

Two times: during tilt and at saturation

Case with $P_m = 1$, $1/\eta = 10^3$ => $\underline{S \approx 1500}$ (as $S = LV_A/\eta$ is a posteriori estimated)



J_{max}

Agreement with tilt theory (linear stability)
Characteristic time for tilt growth: $t_{tilt} \approx 0.5\, t_A \approx L/V_A$

in units of t_A

Results

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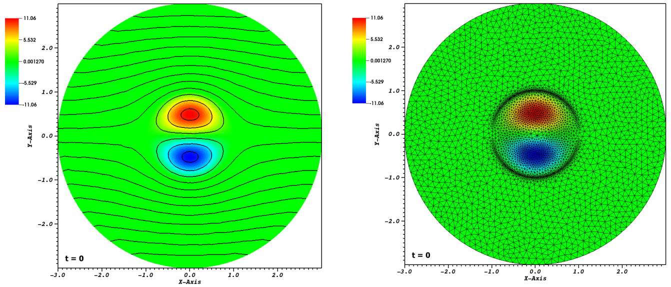

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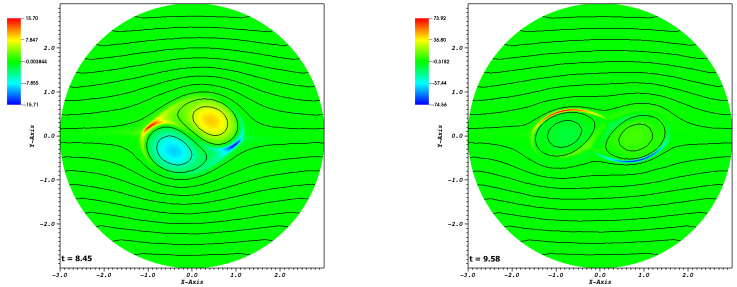

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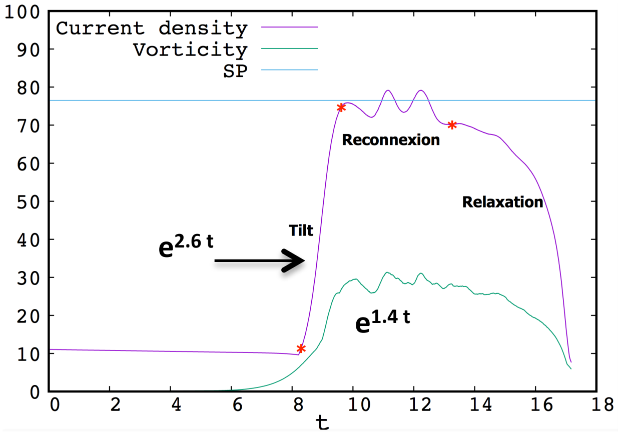
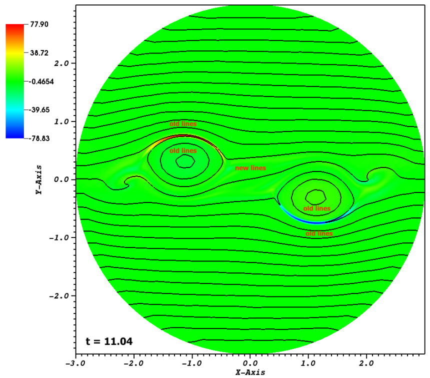
Case with $P_m = 1$, $1/\eta = 10^3$ => $S \approx 1500$

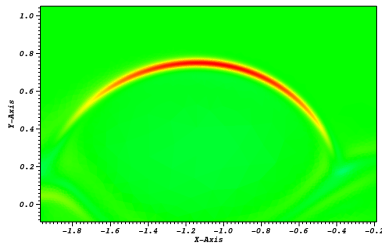


J_{max}

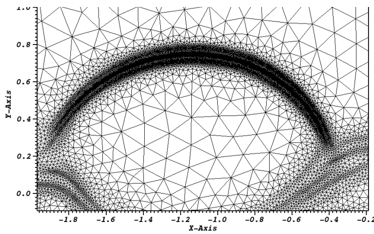
in units of t_A



During reconnection

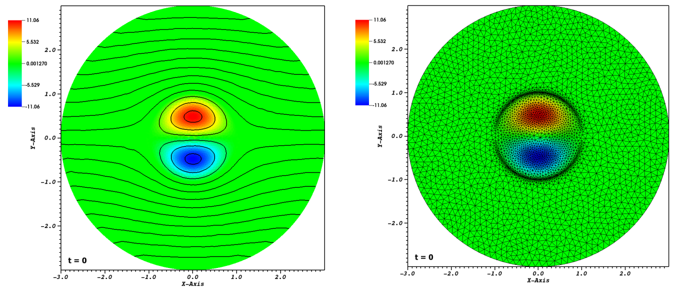


Zoom-in on current layer

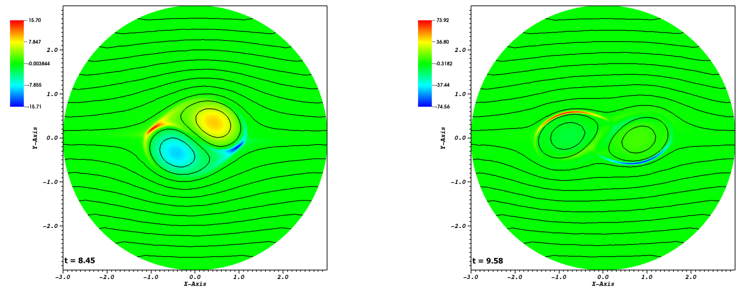


Results

- **Magnetic reconnection with plasmoids during Tilt instability - <u>SP regime</u>**



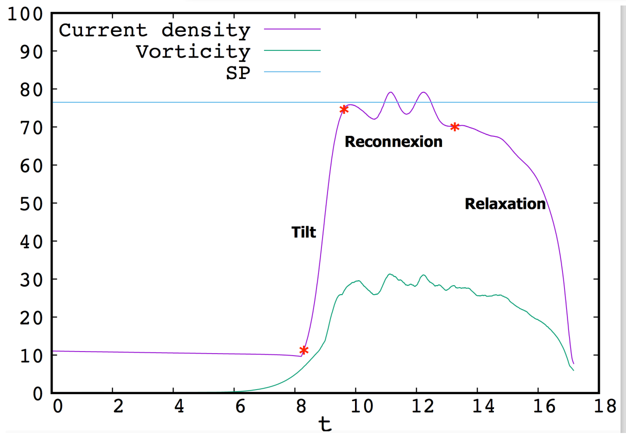
Initial setup



Two times: during tilt and at saturation

J_{max} scales as $S^{1/2}$ and reconnection rate $\eta\, J_{max}$ scaling as $S^{-1/2}$ in agreement with SP process if $S < S_c$

P_m = 1 (fixed)



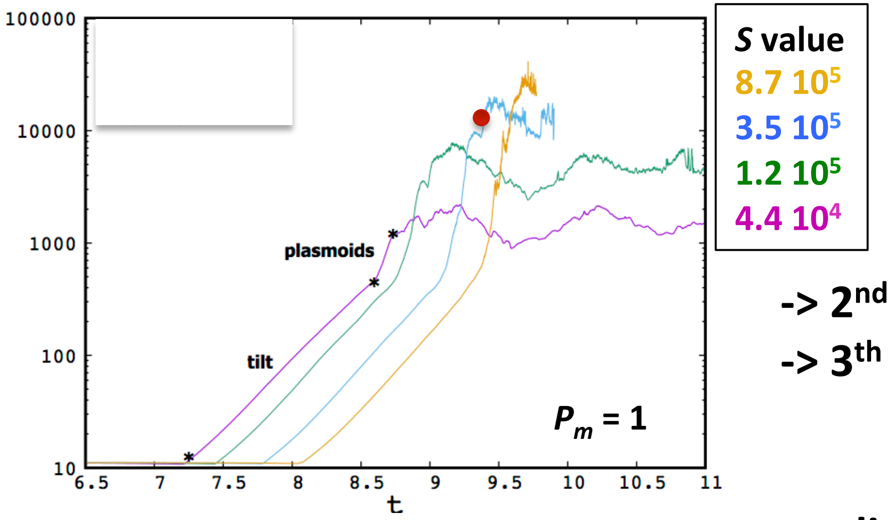
J_{max}

in units of t_A

Results

- **Magnetic reconnection with plasmoids during Tilt instability - <u>Plasmoid regime</u>**

<u>Maximum current density at different *S*</u>

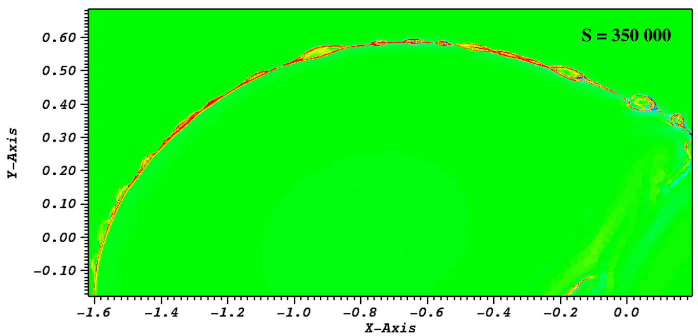


Plasmoids appear for $S \geq 5 \times 10^3$ in this study !

(for coalescence $S_c \approx 3 \times 10^4$)

-> 2nd asterisk: plasmoids visible in the current layer
-> 3th asterisk: plasmoids fully break up the layer followed by stochastic reconnection

qualitatively agree with Comisso theory: quiescent phase followed by an explosive phase with a super-Alfvénic growth is predicted

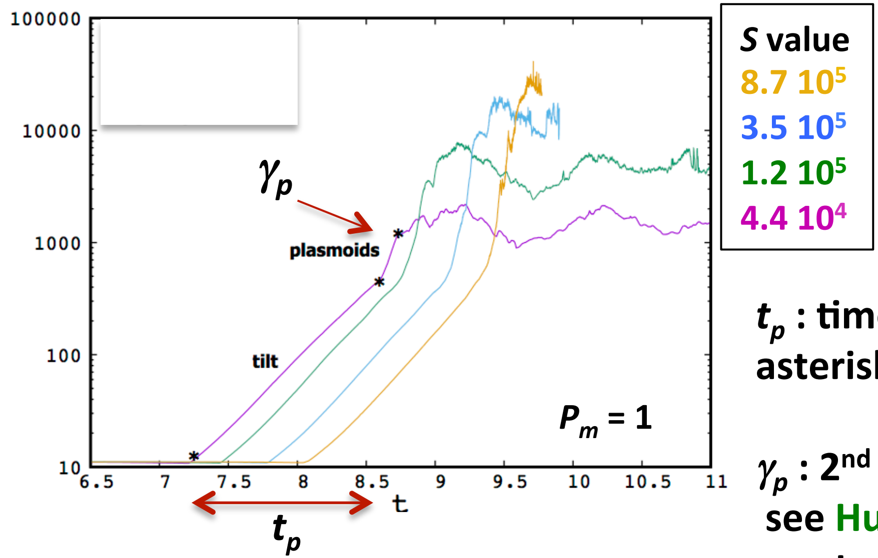


(Saturated value to 300) colored contour map of one (zoom-in) current sheet at disruption time:

Results

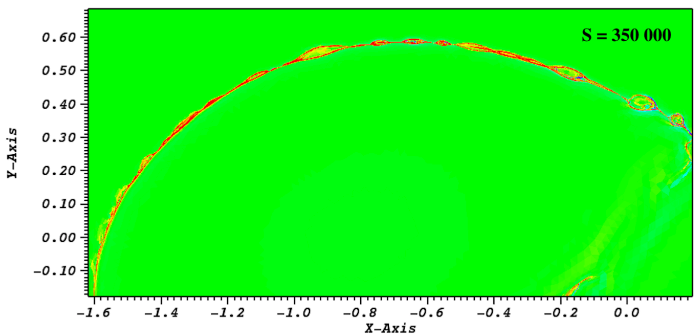
- **Magnetic reconnection with plasmoids during Tilt instability - Plasmoid regime**

Maximum current density at different *S*



t_p : time delay for first plasmoids to appear (between 2 first asterisks)

γ_p : 2nd slope -> interpreted as a dominant mode growth rate see Huang et al. 2017 using coalescence mode, phase related to plasmoids width ~ inner resistive layer width

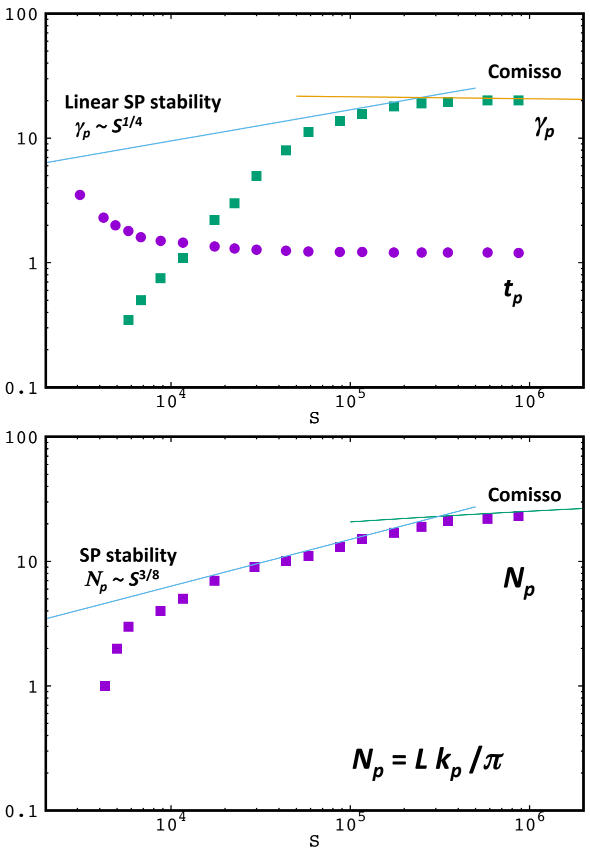


(Saturated) Colored contour map of one (zoom-in) current sheet

Results

- **Magnetic reconnection with plasmoids during Tilt instability - <u>Plasmoid regime</u>**

<u>Plasmoid growth at different *S*</u>



t_p -> 1.2 t_A ~ t_{tilt}
- agree with Udzensky & Loureiro 2016, Tolman et al. 2018
- agree with simulations of Huang et al. 2017

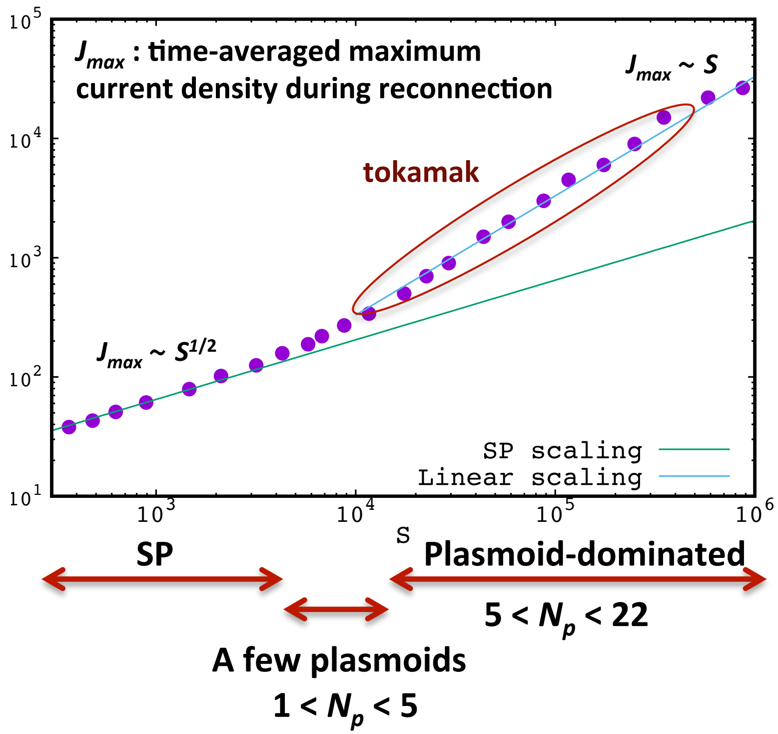
$\gamma_p \sim S^{1/4}$ valid only for intermediate *S*
and saturates with value of order 20 t_A^{-1} for highest *S*
- partly agree with Comisso et al. 2016
- mostly agree with Huang et al. 2017
- disagree with Pucci & Velli 2014

Number of plasmoids at saturation, N_p , scales as $S^{3/8}$ for intermediate *S* and tends to saturate for highest *S*
- partly agree with Comisso et al. 2016
- mostly agree with Huang et al. 2017

smaller values for small S -> outflow reconnection effect !

Results

- **Magnetic reconnection with/without plasmoids during Tilt instability**



FKR modes <-> Coppi modes ?

The normalized <u>reconnection rate</u> in the plasmoid-dominated regime is estimated as:

$\eta J_{max} / (V_A B_u) \approx 0.014$ independent of S that is two times higher than for coalescence
Huang et al. 2017

It can be hardly explained by the fractal (heuristic) model with hierarchical plasmoid chains requiring $N \sim L/L_c \sim S/S_c >> N_p$ (L_c is the smallest marginally stable critical layer length)
Huang & Bhattacharjee 2010, Uzdensky et al. 2010

-> to be explored with longer time simulations !

Conclusions and perspectives

- **Tilt instability is an interesting setup to study the formation of plasmoids & reconnection**

-> Transition between Sweet-Parker and plasmoid regime at $S_c \approx 5 \times 10^3$ (for coalescence $S_c \approx 3 \times 10^4$)

-> Results on plasmoids growth have many similarities with simulations using coalescence setup: super-Alfvénic growth rate ~ 10 - 20 t_A^{-1} following a quiescence phase with $t_p \approx t_{tilt}$, but higher S need to be explored (differences at high S)

-> Results partly agree with the theory of Comisso et al., with the non-power laws with S (differences due to ouflow effect at low S close to S_c)

-> Results different from theory of Velli and coll. (growth rate remain Alfvénic) -> effect of the initial setup subject to ideal versus resistive instabilities ?

- **Perspectives using tilt instability**

-> MHD plasmoid-dominated regime (longer time simulations are required) -> explain reco. rate !

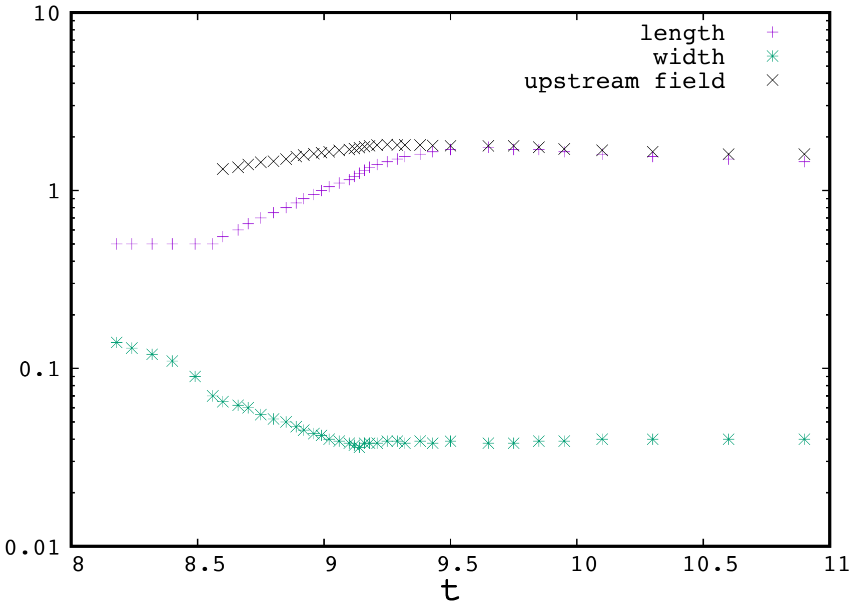
-> Higher S values are necessary for solar corona applications

-> Beyond MHD: for $S = 10^5$ tokamak, the smallest lengths scale ~ 1 mm ~ kinetic scales

Extra slides

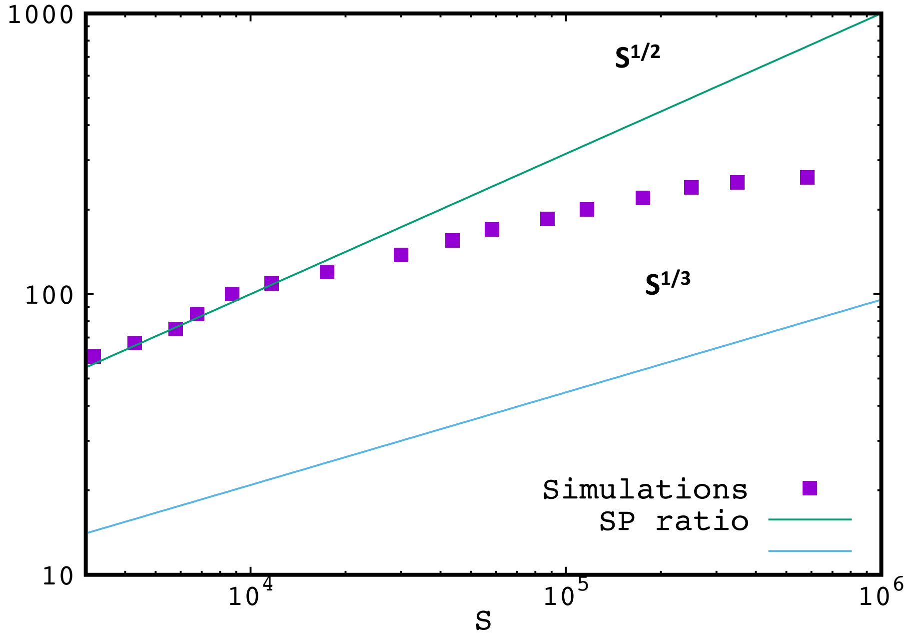
- **Magnetic reconnection with plasmoids !**

Formation of SP current sheet



$S \approx 1500$

Critical aspect ratio before disruption



Extra slides

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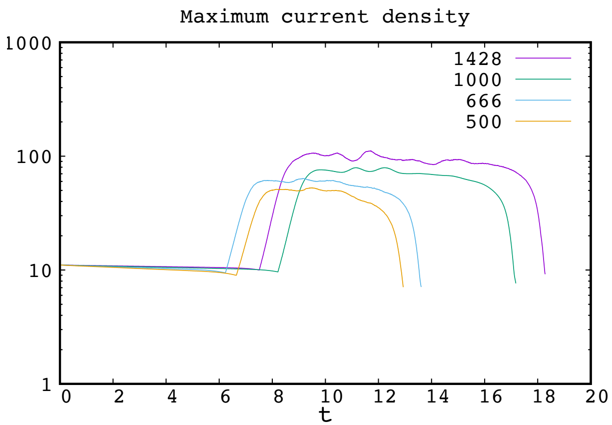
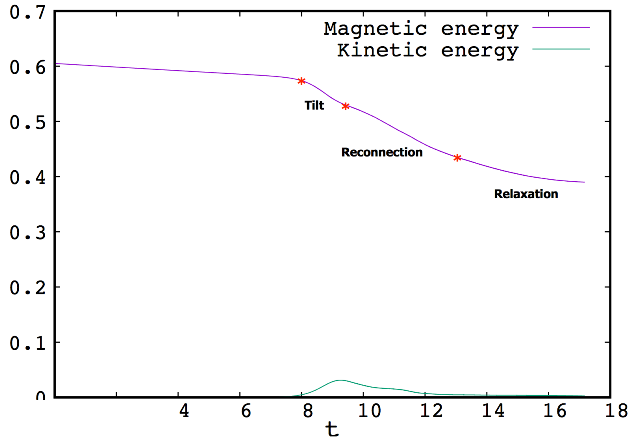
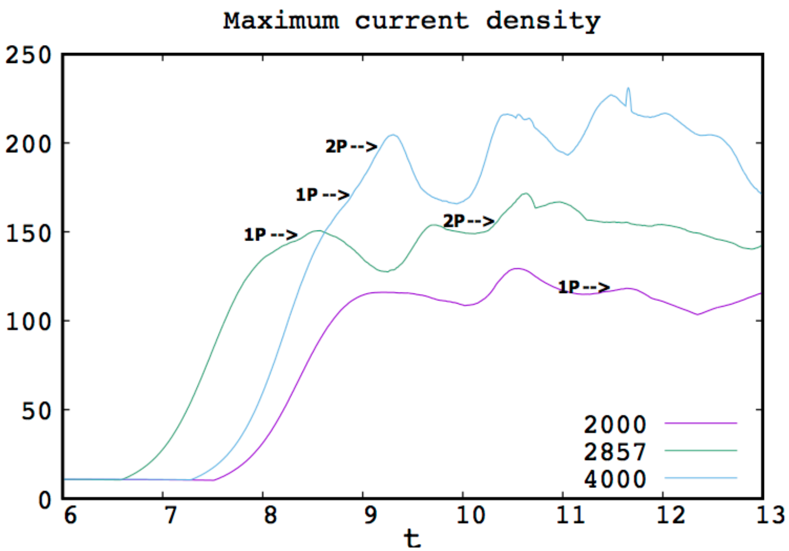


SP regime

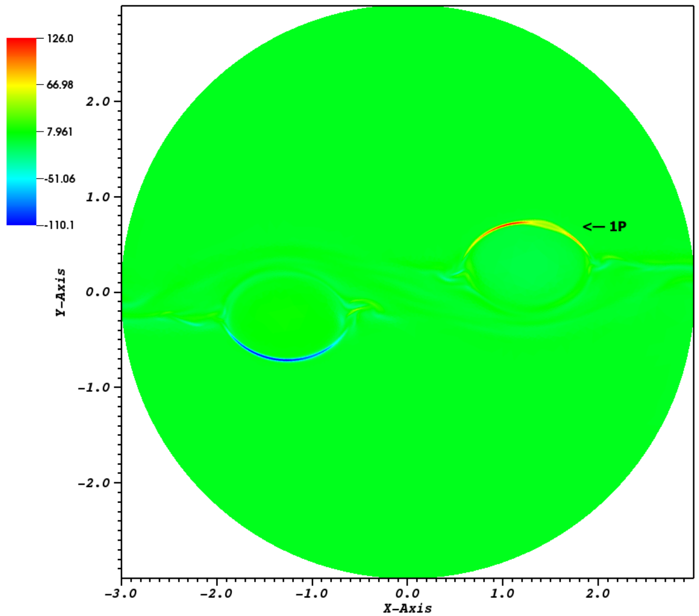


$S \approx 1500$

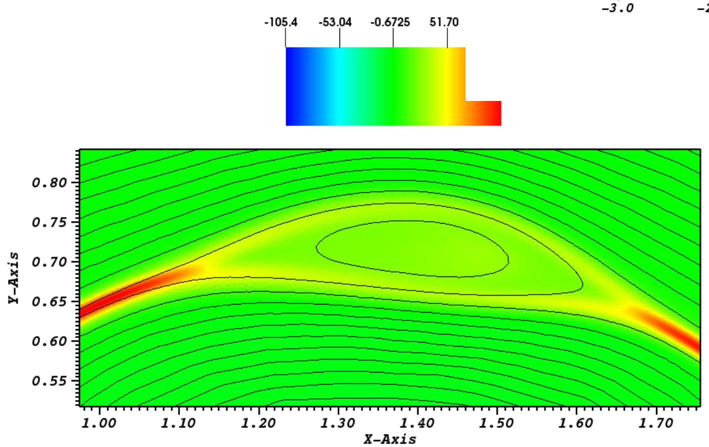


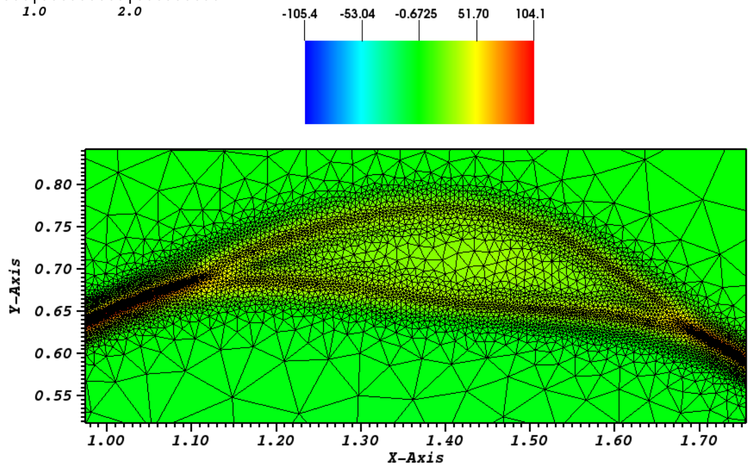
plasmoids regime

Extra slides



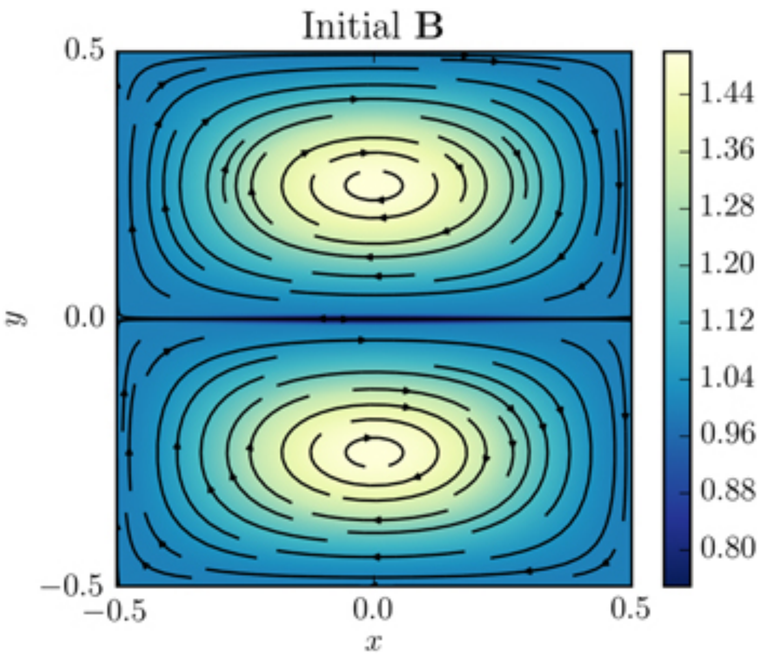
S **= 2000**
(a very late plasmoid)

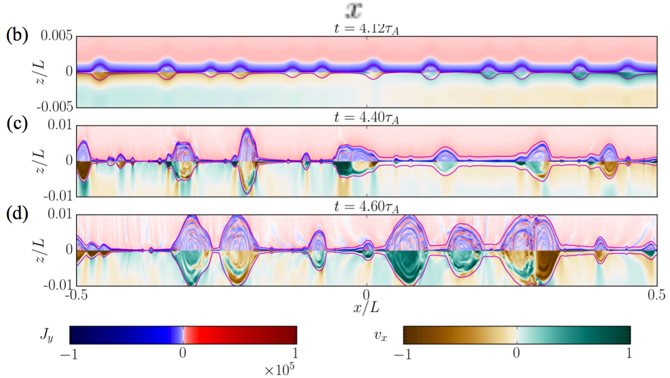




Extra slides

- **Magnetic reconnection with plasmoids !**



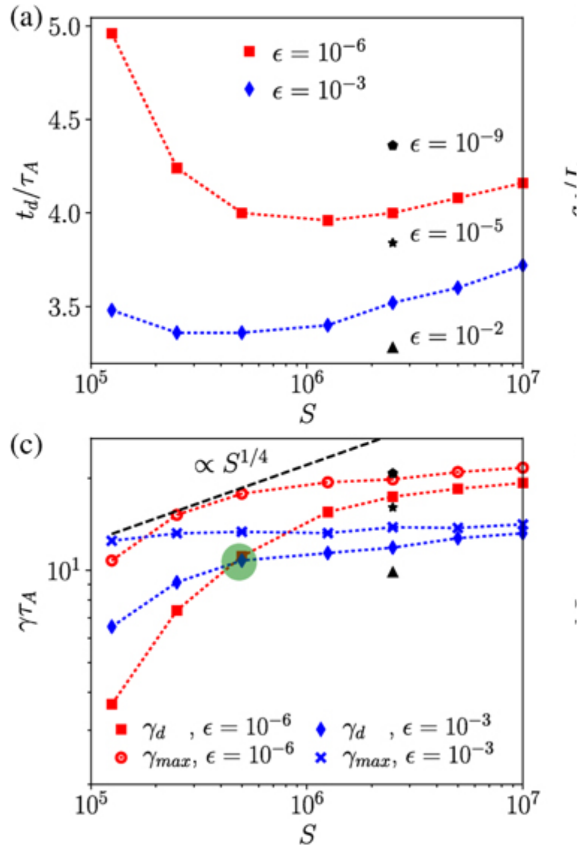


current layer

Numerical simulations Huang et al. 2017

'Standard setup' coalescence instability in 2D between 2 attracting currents

(nearly incompressible)



Agreement with Comisso theory ?